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A corrigendum to "A note on compact-like semitopological groups" [Carpathian Math. Publ. 2019, 11 (2), 442–452]

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Recall that a topological space X is *weakly semiregular*, if X has a base consisting of regular open sets, that is such sets U that $U = \operatorname{int}\overline{U}$. In [1] is stated the following result.

Lemma 3 ([1]). *Let (X,τ) be a weakly semiregular space, (Y,σ) be a space and $\pi : X \to Y$ be a continuous clopen surjection. Then Y is a weakly semiregular space.*

Unfortunately, lemma's proof from [1] contains an error. Namely, the inclusion $\pi(\overline{U}) \subset \pi\pi^{-1}(V)$ fails, for instance, when π is the identity map and $U = V$ is any regular open set such that $U \neq \overline{U}$.

Fortunately, in the paper [1] Lemma 3 is applied only once, namely in conjunction with Lemma 1 to prove Proposition 2. This application can be fixed because the map π considered in Lemma 1 satisfies a condition $\pi^{-1}(\pi(U)) = U$ for every regular open subset of X. Adding this condition to Lemma 3, we can derive the required conclusion as follows.

Let $y \in Y$ be any point and $V \in \sigma$ be any open neighborhood of y. Pick a point $x \in \pi^{-1}(y)$. Since $\pi^{-1}(V)$ is an open neighborhood of x and X is weakly semiregular, there exists a regular open subset U of X such that $x \in U \subset \pi^{-1}(V)$. Since the mapping π is continuous and clopen, we have $\pi(\overline{U}) = \overline{\pi(U)}$ and $\pi(U)$ is open in Y. Since U is open, the set $X \setminus \overline{U}$ is a regular open subset of X. Then $X \setminus \overline{U} = \pi^{-1}(\pi(X \setminus \overline{U}))$. Thus $\overline{U} = \pi^{-1}(\overline{\pi(U)})$.

Suppose that there exists a point $z \in \operatorname{int}\overline{\pi(U)} \setminus V$. Then $\pi^{-1}(z) \subset X \setminus \pi^{-1}(V)$ and $\pi^{-1}(z) \subset \pi^{-1}(\operatorname{int}\overline{\pi(U)}) \subset \pi^{-1}(\pi(\overline{U})) = \overline{U}$. This contradicts with $\operatorname{int}\overline{U} \subset \pi^{-1}(V)$. Thus $y \in \operatorname{int}\overline{\pi(U)} \subset V$, and hence Y is a weakly semiregular space.

References

[1] Ravsky A. *A note on compact-like semitopological groups*. Carpathian Math. Publ. 2019, **11** (2), 442–452. doi:10.15330/cmp.11.2.442-452

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