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On generalized double almost statistical convergence of weight g

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The purpose of this paper is to introduce the concept of λ-double almost statistical convergence of weight g, which emerges naturally from the concept of the double almost convergence and λ-statistical convergence. Some interesting inclusion relations have been considered.

Key words and phrases: weight function g, double statistical convergence, double almost convergence, modulus function.

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Introduction

An extension of the usual concept of sequential limits, which is called statistical convergence, was first recognized by H. Fast [6] as follows.

A sequence (x_k) of real numbers is said to be statistically convergent to L if for an arbitrary $\varepsilon > 0$ we have

$$\lim_{n\to\infty} \frac{1}{n}\big|\{k \leq n : |x_k - L| \geq \varepsilon\}\big| = 0.$$

Statistical convergence turned out to be one of the most active areas of research in summability theory after the works of J.A. Fridy [7], T. Šalát [18], J.S. Connor [5] and some others.

M. Mursaleen [13] defined λ-statistical convergence which is more general than statistical convergence as follows.

A sequence (x_k) is said to be λ-statistically convergent if there is a complex number L such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n}\big|\{k \in I_n : |x_k - L| \geq \varepsilon\}\big| = 0.$$

Later on E. Savaş [19] continued the study of the concept of λ-almost statistical convergence by using almost convergence. Recently, λ-statistical convergence of order α, $0 < \alpha \leq 1$, was introduced and studied by R. Çolak and Ç.A. Bektaş [3]. This is a generalization of λ-statistical convergence.

In this paper, as new and more general approach, we introduce and study the concept of λ-double almost statistical convergence of weight g, where $g : [0,\infty) \times [0,\infty) \to [0,\infty)$, $g(x_{nm}) \to \infty$ for any sequence (x_{nm}) in $[0,\infty) \times [0,\infty)$ with $x_{nm} \to \infty$. Throughout the paper, the class of all such functions will be denoted by $\mathbf{G}$.

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1 Basic facts and definitions

Let w_2 be the class of all real or complex double sequences. By the convergence of a double sequence we mean the convergence in Pringsheim's sense, that is, double sequence $x = (x_{kl})$ has a Pringsheim limit L denoted by P-lim x provided that for a given $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that $|x_{kl} - L| < \epsilon$ whenever $k, l \geq N$. We call such an x more briefly as "P-convergent" (see [15]). Also double sequences were introduced and studied by R.F. Patterson (see [16], [17]) and many others.

We use symbol c_2 to denote the class of P-convergent sequences. A double sequence $x = (x_{kl})$ is bounded if $\|x\| = \sup_{k,l\geq 0} |x_{kl}| < \infty$. Let l_2^∞ and c_2^∞ be the set of all real or complex bounded double sequences and the set of bounded and convergent double sequences, respectively.

Set

$$x_{kl} = \begin{cases} \max(k,l), & \text{if } \min(k,l) = 0, \\ 0, & \text{otherwise.} \end{cases}$$

It is easy to find that $\lim_{k,l} x_{kl} = 0$ but $\sup_{k,l} |x_{kl}| = \infty$. This shows that the convergence of a double sequence in Pringsheim's sense does not imply the boundedness of its terms. Further J.D. Hill [8] studied the double sequences certain results obtained by G.G. Lorentz [9] for single sequences.

Following S. Banach [1] we can easily define the following.

A linear functional φ on l_2^∞ is said to be Banach limit if it has the following properties:

1) $\varphi(x) \geq 0$ if $x \geq 0$, i.e. $x_{kl} \geq 0$ for all k, l;

2) $\varphi(e) = 1$, where $e = (e_{kl})$ with $e_{kl} = 1$ for all k, l;

3) $\varphi(x) = \varphi(S_{10}x) = \varphi(S_{01}x) = \varphi(S_{11}x)$, where the shift operators $S_{10}x$, $S_{01}x$, $S_{11}x$ are defined by $S_{10}x = (x_{k+1,l})$, $S_{01}x = (x_{k,l+1})$, $S_{11}x = (x_{k+1,l+1})$.

Let B_2 be the set of all Banach limits on l_2^∞. A double sequence $x = (x_{kl})$ is said to be almost convergent to a number L if $\varphi(x) = L$ for all $\varphi \in B_2$ (see [8]).

F. Móricz and B.E. Rhoades [11] defined the almost convergence of double sequence as follows.

A double sequence $x = (x_{kl})$ is said to be almost convergent to a number L if

$$P\text{-}\lim_{p,q\to\infty} \sup_{m,n\geq 0} \left| \frac{1}{(p+1)(q+1)} \sum_{k=m}^{m+p} \sum_{l=n}^{n+q} x_{kl} - L \right| = 0,$$

that is, the average value of (x_{ij}) taken over any rectangle

$$D = \{(i,j) : m \leq i \leq m+p, n \leq j \leq n+q\}$$

tends to L as both p and q tend to ∞ and this convergence is uniform in m and n. We denote the space of double almost convergent sequences by $\hat{c}_2$, namely

$$\hat{c}_2 = \left\{ x = (x_{kl}) : \lim_{kl\to\infty} \left| t_{klpq}(x) - L \right| = 0 \text{ uniformly in } p, q \right\},$$

where

$$t_{klpq}(x) = \frac{1}{(k+1)(l+1)} \sum_{k=p}^{k+p} \sum_{l=q}^{l+q} x_{kl}.$$

M. Mursaleen and O.H. Edely [14] presented the notion of statistical convergence for double sequence $x = (x_{kl})$ as follows.

A real double sequence $x = (x_{kl})$ is said to be statistically convergent to L, provided that for each $\varepsilon > 0$

$$P\text{-}\lim_{m,n} \frac{1}{mn} \left|\{(k,l) : k \leq m \text{ and } l \leq n, |x_{kl} - L| \geq \varepsilon\}\right| = 0.$$

More recent developments on double sequences can be found in [2, 4, 10, 12] and some others, where some more references can be found.

Definition 1. *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_m)$ be two non-decreasing sequences of positive real numbers both tending to ∞ as n and m approach ∞, respectively. Also let $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$ and $\mu_{m+1} \leq \mu_m + 1$, $\mu_1 = 1$. We write the generalized double de la Valèe-Poussin mean by*

$$t_{nm}(x) = \frac{1}{\lambda_n \mu_m} \sum_{i \in I_n, j \in I_m} x_{kl}.$$

A sequence $x = (x_{kl})$ is said to be (V^2, λ, μ)-summable to a number L if $t_{nm}(x) \to L$ as $n, m \to \infty$ in Pringsheim's sense.

Throughout this paper, we shall denote $\lambda_n \mu_m$ by $\bar{\lambda}_{nm}$, and $i \in I_n$, $j \in I_m$ by $(i,j) \in I_{nm}$.

2 Main Results

We now introduce our fundamental definition. Throughout this paper, for typographical convenience we shall use the notation x_{klpq} to denote $x_{k+p,l+q}$.

Definition 2. *Let the sequence $\lambda = (\lambda_{nm})$ of real numbers be defined as above and let $g \in \mathbf{G}$. A sequence $x = (x_{kl})$ is said to be λ-double almost statistically convergent of weight g if there is a complex number L such that*

$$P\text{-}\lim_{mn \to \infty} \frac{1}{g(\bar{\lambda}_{nm})} \left|\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\}\right| = 0$$

uniformly in p, q. In this case we write $\hat{S}^g_\lambda$-$\lim x_{kl} = L$.

The set of all λ-double almost statistically convergent sequences of weight g will be denoted by $\hat{S}^g_\lambda$. For example, the sequence $x = (x_{kl})$ defined by

$$x_{klpq} = \begin{cases} klpq, & klpq = (nm)^2, \\ 0, & klpq \neq (nm)^2, \end{cases} \quad n, m = 1, 2, \ldots,$$

is λ-double almost statistically convergent of weight g to 0 for any $g \in \mathbf{G}$, for which there exist $M_1, M_2 > 0$ and $(r,s) \in \mathbb{N} \times \mathbb{N}$ such that

$$M_1 \leq \frac{(nm)^\alpha}{g(nm)} \leq M_2 \text{ for all } n \geq r \text{ and } m \geq s,$$

where $\frac{1}{2} < \alpha \leq 1$ and $\lambda = (nm)$.

Remark 1. *In the above definition, if we consider $g(\lambda_{nm}) = (nm)$, $\alpha = 1$, we have the notion of double almost statistical convergence [8]. The set of all double almost statistically convergent sequences will be denoted by $\hat{S}$.*

This definition led to the following theorem.

Theorem 1. *Let $g \in \mathbf{G}$ and $x = (x_{kl})$, $y = (y_{kl})$ be sequences of complex numbers.*

(i) If $\hat{S}_\lambda^g$-$\lim x_{kl} = x_0$ and $c \in \mathbb{C}$, then $\hat{S}_\lambda^g$-$\lim cx_{kl} = cx_0$.

(ii) If $\hat{S}_\lambda^g$-$\lim x_{kl} = x_0$ and $\hat{S}_\lambda^g$-$\lim y_{kl} = y_0$, then $\hat{S}_\lambda^g$-$\lim(x_{kl} + y_{kl}) = x_0 + y_0$.

Proof. *(i)* For $c = 0$ the result is clear. Let $c \neq 0$. We find that

$$\frac{1}{g(\bar{\lambda}_{nm})} \left|\left\{(k,l) \in I_{nm} : \left|cx_{klpq} - cx_0\right| \geq \varepsilon\right\}\right| = \frac{1}{g(\bar{\lambda}_{nm})} \left|\left\{(k,l) \in I_{nm} : \left|x_{klpq} - x_0\right| \geq \frac{\varepsilon}{|c|}\right\}\right|$$

and the result follows.

(ii) The result follows from the fact that

$$\begin{aligned}\frac{1}{g(\bar{\lambda}_{nm})}&\left|\left\{(k,l) \in I_{nm} : |x_{klpq} + y_{klpq} - (x_0 + y_0)| \geq \varepsilon\right\}\right| \\ &\leq \frac{1}{g(\bar{\lambda}_{nm})} \left|\left\{(k,l) \in I_{nm} : \left|x_{klpq} - x_0\right| \geq \frac{\varepsilon}{2}\right\}\right| \\ &\quad + \frac{1}{g(\bar{\lambda}_{nm})} \left|\left\{(k,l) \in I_{nm} : \left|y_{klpq} - y_0\right| \geq \frac{\varepsilon}{2}\right\}\right|.\end{aligned}$$

□

Definition 3. *Let $\lambda = (\lambda_{nm})$ be as above and let $g \in \mathbf{G}$. Let t be a positive real number. A sequence $x = (x_{kl})$ is said to be strongly $(\hat{V}, \lambda)$-double almost summable of weight g if there is a complex number L such that*

$$\lim_{n,m\to\infty} \frac{1}{g(\bar{\lambda}_{nm})} \sum_{(k,l)\in I_{nm}} \left|x_{klpq} - L\right|^t = 0$$

uniformly in p, q. The set of all strongly $(\hat{V}, \lambda)$-double almost summable sequences of weight g will be denoted by $\left[\hat{V}_t^g, \lambda\right]$.

Remark 2. *For $g(n) = (nm)^\alpha$, $0 < \alpha \leq 1$, this notion coincides with the notion of strong $(\hat{V}, \lambda)$-double almost summability of order α.*

Theorem 2. *Let $g_1, g_2 \in \mathbf{G}$. If there exist $M > 0$ and $(r,s) \in \mathbb{N} \times \mathbb{N}$ such that $g_1(\lambda_{nm})/g_2(\bar{\lambda}_{nm}) \leq M$ for all $n \geq r$ and $m \geq s$, then $\hat{S}_\lambda^{g_1} \subseteq \hat{S}_{\lambda,}^{g_2}$.*

Proof. Write that,

$$\begin{aligned}\frac{1}{g_2(\bar{\lambda}_{nm})} \left|\left\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\right\}\right| &= \frac{g_1(\bar{\lambda}_{nm})}{g_2(\bar{\lambda}_{nm})} \cdot \frac{1}{g_1(\bar{\lambda}_{nm})} \left|\left\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\right\}\right| \\ &\leq M \cdot \frac{1}{g_1(\bar{\lambda}_{nm})} \left|\left\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\right\}\right|\end{aligned}$$

for all $n \geq r$ and $m \geq s$. If $x = (x_{kl}) \in \hat{S}_{\lambda}^{g_1}$, then the right hand side tends to zero uniformly in p, q for every $\varepsilon > 0$ and in this case

$$\frac{1}{g_2(\bar{\lambda}_{nm})} \left| \{(k,l) \in I_{nm} : \left| x_{klpq} - L \right| \geq \varepsilon\} \right| = 0$$

uniformly in p, q and finally $x \in \hat{S}_{\lambda}^{g_2}$. Hence $\hat{S}_{\lambda}^{g_1} \subseteq \hat{S}_{\lambda}^{g_2}$. □

Corollary 1. *In particular, let* $g \in \boldsymbol{G}$ *and if there exist* $M > 0$ *and* $(r,s) \in \mathbb{N} \times \mathbb{N}$ *such that* $(nm)/g(\bar{\lambda}_{nm}) \leq M$ *for all* $n \geq r$ *and* $m \geq s$, *then* $\hat{S}_{\lambda}^{g} \subseteq \hat{S}_{\lambda}$.

Theorem 3. $\hat{S} \subseteq \hat{S}_{\lambda}^{g}$ *if* $\liminf_{nm \to \infty} \dfrac{g(\bar{\lambda}_{nm})}{(nm)} > 0$.

Proof. For any $\varepsilon > 0$, we write

$$\left\{k \leq n \text{ and } l \leq m : \left| x_{klpq} - L \right| \geq \varepsilon \right\} \supseteq \left\{(k,l) \in I_{nm} : \left| x_{klpq} - L \right| \geq \varepsilon \right\}.$$

Hence, it follows that for $p, q \in \mathbb{N}$

$$\begin{aligned} \frac{1}{nm} \left| \left\{k \leq n \text{ and } l \leq m : \left| x_{klpq} - L \right| \geq \varepsilon \right\} \right| &\geq \frac{1}{nm} \left| \left\{(k,l) \in I_{nm} : \left| x_{klpq} - L \right| \geq \varepsilon \right\} \right| \\ &\geq \frac{g(\bar{\lambda}_{nm})}{nm} \cdot \frac{1}{g(\bar{\lambda}_{nm})} \left| \left\{(k,l) \in I_{nm} : \left| x_{klpq} - L \right| \geq \varepsilon \right\} \right|. \end{aligned}$$

If $x \to L(\hat{S})$, then $\frac{1}{nm} \left| \left\{k \leq n \text{ and } l \leq m : \left| x_{klpq} - L \right| \geq \varepsilon \right\} \right| \to 0$ as $n, m \to \infty$ and consequently we find

$$\frac{1}{nm} \left| \left\{k \leq n \ \text{ and } \ l \leq m : \left| x_{klpq} - L \right| \geq \varepsilon \right\} \right| \to 0$$

and so

$$\frac{1}{g(\bar{\lambda}_{nm})} \left| \left\{(k,l) \in I_{nm} : \left| x_{klpq} - L \right| \geq \varepsilon \right\} \right| \to 0$$

as $n, m \to \infty$. It is clear that $x \to L(\hat{S}_{\lambda}^{g})$. □

Theorem 4. *Let* $g_1, g_2 \in \boldsymbol{G}$. *If there exist* $M > 0$ *and* $(r,s) \in \mathbb{N} \times \mathbb{N}$ *such that*

$$g_1(\bar{\lambda}_{nm}) / g_2(\bar{\lambda}_{nm}) \leq M$$

for all $n \geq r$ *and* $m \geq s$, *then* $\left[\hat{V}_t^{g_1}, \lambda\right] \subseteq \left[\hat{V}_t^{g_2}, \lambda\right]$.

Proof. The proof is similar to the proof of Theorem 3.6 and so is omitted. □

Corollary 2. *Let* $g \in \boldsymbol{G}$. *If there exist* $M > 0$ *and* $(r,s) \in \mathbb{N} \times \mathbb{N}$ *such that* $(nm)/g(\lambda_{nm}) \leq M$ *for all* $n \geq r$ *and* $m \geq s$, *then* $\hat{S}_{\lambda}^{g} \subseteq \hat{S}_{\lambda}$.

Theorem 5. *If* $0 < t < u < \infty$ *and* $g \in \boldsymbol{G}$, *then* $\left[\hat{V}_u^{g}, \lambda\right] \subset \left[\hat{V}_t^{g}, \lambda\right]$.

The proof follows from Hölder's inequality.

Theorem 6. *Let $g_1, g_2 \in \boldsymbol{G}$ and there exist $M > 0$ and $(r,s) \in \mathbb{N} \times \mathbb{N}$ such that $g_1(\bar{\lambda}_{nm})/g_2(\bar{\lambda}_{nm}) \leq M$ for all $n \geq r$ and $m \geq s$ and let $0 < p < \infty$. If a sequence $x = (x_{kl})$ is strongly $(\hat{V}, \lambda)$-almost double summable of weight g_1 to L, then it is λ-almost double statistically convergent of weight g_2 to L, i.e $\left[\hat{V}_t^{g_1}, \lambda\right] \subset \hat{S}_\lambda^{g_2}$.*

Proof. Let $x = (x_{kl}) \in \left[\hat{V}_p^{g_1}, \lambda\right]$ and let $\varepsilon > 0$ be given. Consider

$$\begin{aligned}\sum_{(k,l)\in I_{nm}} \left|x_{klpq} - L\right|^t &= \sum_{\substack{(k,l)\in I_{nm}\\ |x_{klpq}-L|\geq\varepsilon}} \left|x_{klpq} - L\right|^t + \sum_{\substack{(k,l)\in I_{nm}\\ |x_{klpq}-L|<\varepsilon}} \left|x_{klpq} - L\right|^t \\ &\geq \sum_{\substack{k\in I_n\\ |x_{klpq}-L|\geq\varepsilon}} \left|x_{klpq} - L\right|^p \geq \left|\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\}\right| \cdot \varepsilon^t.\end{aligned}$$

Now it follows that

$$\begin{aligned}\frac{1}{g_1(\bar{\lambda}_{nm})} \sum_{(k,l)\in I_{nm}} \left|x_{klpq} - L\right|^t &\geq \frac{1}{g_1(\bar{\lambda}_{nm})} \left|\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\}\right| \cdot \varepsilon^t \\ &= \frac{g_2(\bar{\lambda}_{nm})}{g_1(\bar{\lambda}_{nm})} \cdot \frac{1}{g_2(\bar{\lambda}_{nm})} \left|\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\}\right| \cdot \varepsilon^t \\ &\geq \frac{1}{M} \cdot \frac{1}{g_2(\bar{\lambda}_{nm})} \left|\{(k,l) \in I_{nm} : \left|x_{klpq} - L\right| \geq \varepsilon\}\right| \cdot \varepsilon^t\end{aligned}$$

for all $n \geq r$ and $m \geq s$. If $x \to L(\left[\hat{V}_t^{g_1}, \lambda\right])$ then the left hand side tends to zero and consequently the right hand side also tends to zero uniformly in p, q. Hence $x \to L(\hat{S}_\lambda^{g_2})$. ☐

Corollary 3. *Let $g \in \boldsymbol{G}$. If there exist $M > 0$ and $(r,s) \in N \times N$ such that $\frac{nm}{g(\lambda_n m)} \leq M$ for all $n \geq r, m \geq s$ and $0 < p < \infty$, then $\left[\hat{V}_t^g, \lambda\right] \subseteq \hat{S}_\lambda$.*

3 Conclusion

Recently, λ-statistical convergence has been considered as a better option than statistically convergence. It is found very interesting that some results on sequences, series and summability can be proved by replacing the statistical convergence by λ-statistical convergence. This concept has also been defined and studied in different setups. In this paper, we study the concept of λ-double almost statistical convergence of weight g, which emerges naturally from the concepts of the double almost convergence and λ-double statistical convergence.

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Метою цієї статті є впровадити поняття λ-подвійної майже статистичної збіжності з вагою g, яка природним чином випливає з поняття подвійної майже збіжності та λ-статистичної збіжності. У статті розглянуто деякі цікаві відношення включення.

Ключові слова і фрази: вагова функція g, подвійна статистична збіжність, подвійна майже збіжність, модуль функції.