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Dynamical behavior of one rational fifth-order difference equation

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In this paper, we study the qualitative behavior of the rational recursive equation

$$x_{n+1} = \frac{x_{n-4}}{\pm 1 \pm x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}, \quad n \in \mathbb{N}_0 := \{0\} \cup \mathbb{N},$$

where the initial conditions are arbitrary nonzero real numbers. The main goal of this paper, is to obtain the forms of the solutions of the nonlinear fifth-order difference equations, where the initial conditions are arbitrary positive real numbers. Moreover, we investigate stability, boundedness, oscillation and the periodic character of these solutions. The results presented in this paper improve and extend some corresponding results in the literature.

Key words and phrases: difference equation, recursive sequence, periodic solution.

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Introduction

Our aim in this paper is to investigate the behavior of the solution of the following nonlinear recursive sequence, defined by

$$x_{n+1} = \frac{x_{n-4}}{\pm 1 \pm x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}, \quad n \in \mathbb{N}_0 := \{0\} \cup \mathbb{N}, \tag{1}$$

where the initial conditions are arbitrary nonzero positive real numbers.

The study and solution of nonlinear rational recursive sequence of high order is quite challenging and rewarding. In the recent times, nonlinear difference equations have a critical role in the fields of pyhsics, economy, ecology, computational science engineering, etc. Many researchers have investigated the behavior of the solution of nonlinear difference equations. So, recently there has been an increasing interest in the study of qualitative analysis of rational difference equations.

R.P. Agarwal et al. [1] studied qualitative behavior solutions of the difference equation

$$x_{n+1} = a + \frac{d x_{n-l} x_{n-k}}{b - c x_{n-s}},$$

where a, b, c, d are positive real constants.

M. Aloqeili [2] has obtained the solutions of the difference equation

$$x_{n+1} = \frac{x_{n-1}}{a - x_n x_{n-1}}.$$

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In [3], A. Bilgin and M.R.S. Kulenović presented some basic discrete models in population dynamics of single species with several age classes. They considered the effect of the constant and periodic immigration and emigration on the global properties of Beverton-Holt model. They also considered the effect of the periodic environment on the global properties of the Beverton-Holt model.

In [4], R. Devault et al. studied the following problem

$$x_{n+1} = \frac{A}{x_n} + \frac{1}{x_{n-2}}, \quad n = 0, 1, 2, \ldots, \quad A \in (0, \infty),$$

and showed every positive solution of the equation.

E.M. Elsayed [5] investigated the solutions of difference equation

$$x_{n+1} = \frac{x_{n-5}}{-1 + x_{n-2}x_{n-5}}.$$

In [6,7], E.M. Elsayed obtained the solutions of the following difference equations

$$x_{n+1} = \frac{x_{n-7}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}}, \qquad x_{n+1} = \frac{x_{n-9}}{\pm 1 \pm x_{n-4}x_{n-9}}.$$

In [8], E.M. Elsayed obtained the solution of the following difference equation

$$x_{n+1} = \frac{x_{n-11}}{\pm 1 \pm x_{n-2}x_{n-5}x_{n-8}x_{n-11}}.$$

A. Gelişken [9] investigated behaviors of well-defined solutions of the following system

$$x_{n+1} = \frac{A_1 y_{n-(3k-1)}}{B_1 + C_1 y_{n-(3k-1)} x_{n-(2k-1)} y_{n-(k-1)}},$$

$$y_{n+1} = \frac{A_2 x_{n-(3k-1)}}{B_2 + C_2 x_{n-(3k-1)} y_{n-(2k-1)} x_{n-(k-1)}},$$

where $n \in \mathbb{N}_0$, $k \in \mathbb{N}_0$, the coefficients $A_1, A_2, B_1, B_2, C_1, C_2$ and the initial conditions are arbitrary real numbers.

R. Karataş et al. [10] gave the solution of the following difference equation

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-2}x_{n-5}}.$$

In [13–15], D. Şimşek et al. studied the following problems with positive initial values

$$x_{n+1} = \frac{x_{n-3}}{1 + x_{n-1}}, \quad x_{n+1} = \frac{x_{n-5}}{1 + x_{n-2}}, \quad x_{n+1} = \frac{x_{n-5}}{1 + x_{n-1}x_{n-3}},$$

for $n = 0, 1, 2, \ldots$.

D. Şimşek et al. [16] gave the solution of difference equation

$$x_{n+1} = \frac{x_{n-(4k+3)}}{1 + \prod_{t=0}^{2} x_{n-(k+1)t-k}},$$

where initial conditions are positive real numbers.

E. Tasdemir [17] studied the global asymptotic stability of the following system of difference equations with quadratic terms

$$x_{n+1} = A + B\frac{y_n}{y_{n-1}^2}, \quad y_{n+1} = A + B\frac{x_n}{x_{n-1}^2},$$

where A and B are positive numbers and the initial values are positive numbers. He also investigated the rate of convergence and oscillation behaviour of the solutions of related system.

D.T. Tollu and I. Yalçinkaya [18] investigated the global behavior of the positive solutions of the system of difference equations

$$u_{n+1} = \frac{a_1 u_{n-1}}{\beta_1 + \gamma_1 + v_{n-2}^p}, \quad v_{n+1} = \frac{a_2 v_{n-1}}{\beta_2 + \gamma_2 + w_{n-2}^q}, \quad w_{n+1} = \frac{a_3 w_{n-1}}{\beta_3 + \gamma_3 + u_{n-2}^r}$$

for $n \in \mathbb{N}_0$, where the initial conditions u_{-i}, v_{-i}, w_{-i} are nonnegative real numbers and the parametrs are positive real numbers.

Let I be some interval of real numbers and let $f : I^{k+1} \to I$ be a continuously differentiable function. Then for every set of initial conditions $x_{-k}, x_{-k+1}, \ldots, x_0 \in I$ the difference equation

$$x_{n+1} = f(x_n, x_{n-1}, \ldots, x_{n-k}), \quad n \in \mathbb{N}_0, \tag{2}$$

has a unique solution $\{x_n\}_{n=-k}^{\infty}$ (see [12]). A point $\overline{x} \in I$ is called an equilibrium point of (2) if

$$\overline{x} = f(\overline{x}, \overline{x}, \ldots, \overline{x}).$$

That is, $x_n = \overline{x}$ for $n \geq 0$ is a solution of (2), or equivalently, $\overline{x}$ is a fixed point of f.

Definition (Stability). *(a) The equilibrium point $\overline{x}$ of* (2) *is called locally stable if for every $\epsilon > 0$, there exists $\delta > 0$ such that for all $x_{-k}, x_{-k+1}, \ldots, x_{-1}, x_0 \in I$ with*

$$|x_{-k} - \overline{x}| + |x_{-k+1} - \overline{x}| + \cdots + |x_0 - \overline{x}| < \delta,$$

we have $|x_n - \overline{x}| < \epsilon$ for all $n \geq k$.

(b) The equilibrium point $\overline{x}$ of (2) *is called locally asymptotically stable if $\overline{x}$ is a locally stable solution of* (2) *and there exists $\gamma > 0$, such that for all $x_{-k}, x_{-k+1}, \ldots, x_{-1}, x_0 \in I$ with*

$$|x_{-k} - \overline{x}| + |x_{-k+1} - \overline{x}| + \cdots + |x_0 - \overline{x}| < \gamma,$$

we have $\lim_{n\to\infty} x_n = \overline{x}$.

(c) The equilibrium point $\overline{x}$ of (2) *is called a global attractor if for all $x_{-k}, x_{-k+1}, \ldots, x_{-1}, x_0 \in I$, we have $\lim_{n\to\infty} x_n = \overline{x}$.*

(d) The equilibrium point $\overline{x}$ of (2) *is called a global asymptotically stable if $\overline{x}$ is locally stable and $\overline{x}$ is also a global attractor of* (2).

(e) The equilibrium point $\overline{x}$ of (2) *is called unstable if $\overline{x}$ is not locally stable.*

The linearized equation of (2) *about the equilibrium $\overline{x}$ is the linear difference equation*

$$y_{n+1} = \sum_{i=0}^{k} \frac{\partial f(\overline{x}, \overline{x}, \ldots, \overline{x})}{\partial x_{n-i}} y_{n-i}.$$

Theorem 1 ([11])**.** *Assume that $p, q \in \mathbb{R}$ and $k \in \mathbb{N}_0$. Then $|p| + |q| < 1$ is a sufficient condition for the asymptotic stability of the difference equation*

$$x_{n+1} + px_n + qx_{n-k} = 0, \quad n \in \mathbb{N}_0.$$

Remark. *Theorem 1 can be easily extended to general linear equation of the form*

$$x_{n+k} + p_1 x_{n+k-1} + \cdots + p_k x_n = 0, \quad n \in \mathbb{N}_0, \tag{3}$$

where $p_1, p_2, \ldots, p_k \in \mathbb{R}$ and $k \in \mathbb{N}$. Then equation (3) *is asymptotically stable provided that*

$$\sum_{i=1}^{k} |p_i| < 1.$$

Definition (Periodicity)**.** *A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be periodic with period p if $x_{n+p} = x_n$ for all $n \geq -k$.*

1 Solution of the Difference Equation $x_{n+1} = \frac{x_{n-4}}{1+x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}$

In this part, we give a specific form of the solutions of the difference equation

$$x_{n+1} = \frac{x_{n-4}}{1 + x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}, \quad n \in \mathbb{N}_0, \tag{4}$$

where the initial conditions are arbitrary nonzero positive real numbers.

Theorem 2. *Let $\{x_n\}_{n=-4}^{\infty}$ be a solution of* (4)*. Then for $n \in \mathbb{N}_0$,*

$$x_{5n-4} = \frac{e \prod_{i=0}^{n-1} (1 + 5iabcde)}{\prod_{i=0}^{n-1} (1 + (5i+1)abcde)}, \qquad x_{5n-3} = \frac{d \prod_{i=0}^{n-1} (1 + (5i+1)abcde)}{\prod_{i=0}^{n-1} (1 + (5i+2)abcde)},$$

$$x_{5n-2} = \frac{c \prod_{i=0}^{n-1} (1 + (5i+2)abcde)}{\prod_{i=0}^{n-1} (1 + (5i+3)abcde)}, \qquad x_{5n-1} = \frac{b \prod_{i=0}^{n-1} (1 + (5i+3)abcde)}{\prod_{i=0}^{n-1} (1 + (5i+4)abcde)},$$

$$x_{5n} = \frac{a \prod_{i=0}^{n-1} (1 + (5i+4)abcde)}{\prod_{i=0}^{n-1} (1 + (5i+5)abcde)},$$

where $x_{-4} = e$, $x_{-3} = d$, $x_{-2} = c$, $x_{-1} = b$, $x_0 = a$.

Proof. For $n = 0$ the result holds. Now suppose that $n > 0$ and that our assumption holds for $n - 1$. That is,

$$x_{5n-9} = \frac{e \prod_{i=0}^{n-2} (1 + 5iabcde)}{\prod_{i=0}^{n-2} (1 + (5i+1)abcde)}, \qquad x_{5n-8} = \frac{d \prod_{i=0}^{n-2} (1 + (5i+1)abcde)}{\prod_{i=0}^{n-2} (1 + (5i+2)abcde)},$$

$$x_{5n-7} = \frac{c \prod_{i=0}^{n-2} (1 + (5i+2)abcde)}{\prod_{i=0}^{n-2} (1 + (5i+3)abcde)}, \qquad x_{5n-6} = \frac{b \prod_{i=0}^{n-2} (1 + (5i+3)abcde)}{\prod_{i=0}^{n-2} (1 + (5i+4)abcde)},$$

$$x_{5n-5} = \frac{a \prod_{i=0}^{n-2} (1 + (5i+4)abcde)}{\prod_{i=0}^{n-2} (1 + (5i+5)abcde)}.$$

Now, it follows from (4) that

$$\begin{aligned}
x_{5n-4} &= \frac{x_{5n-9}}{1+x_{5n-5}x_{5n-6}x_{5n-7}x_{5n-8}x_{5n-9}} \\
&= \frac{\frac{e\prod_{i=0}^{n-2}(1+5iabcde)}{\prod_{i=0}^{n-2}(1+(5i+1)abcde)}}{1+abcde\prod_{i=0}^{n-2}\frac{1+(5i+4)abcde}{1+(5i+5)abcde}\frac{1+(5i+3)abcde}{1+(5i+4)abcde}\frac{1+(5i+2)abcde}{1+(5i+3)abcde}\cdots\frac{1+5iabcde}{1+(5i+1)abcde}} \\
&= \frac{e\prod_{i=0}^{n-2}\frac{(1+5iabcde)}{(1+(5i+1)abcde)}}{1+abcde\prod_{i=0}^{n-2}\frac{1+5iabcde}{1+(5i+5)abcde}} = e\prod_{i=0}^{n-2}\frac{1+5iabcde}{1+(5i+1)abcde}\left(\frac{1}{1+\frac{abcde}{1+(5n-5)abcde}}\right) \\
&= e\prod_{i=0}^{n-2}\frac{1+5iabcde}{1+(5i+1)abcde}\left(\frac{1+(5n-5)abcde}{1+(5n-4)abcde}\right).
\end{aligned}$$

Hence, we have

$$x_{5n-4} = e\prod_{i=0}^{n-1}\frac{1+5iabcde}{1+(5i+1)abcde}.$$

□

Theorem 3. *The equation* (4) *has unique equilibrium point which is the number zero and this equilibrium is not locally asymptotically stable. Also* $\overline{x}$ *is non hyperbolic.*

Proof. For the equilibrium points of (4) we can write

$$\overline{x} = \frac{\overline{x}}{1+\overline{x}^5}.$$

Then $\overline{x}+\overline{x}^6 = \overline{x}$, $\overline{x}^6 = 0$. Thus the equilibrium point of (4) is $\overline{x} = 0$. Let $f:(0,\infty)^5 \to (0,\infty)$ be the function defined by

$$f(l,o,t,w,\alpha) = \frac{l}{1+lotw\alpha}.$$

Therefore it follows that

$$\begin{aligned}
f_l(l,o,t,w,\alpha) &= \frac{1}{(1+lotw\alpha)^2}, & f_o(l,o,t,w,\alpha) &= \frac{-l^2tw\alpha}{(1+lotw\alpha)^2}, \\
f_t(l,o,t,w,\alpha) &= \frac{-l^2ow\alpha}{(1+lotw\alpha)^2}, & f_w(l,o,t,w,\alpha) &= \frac{-l^2ot\alpha}{(1+lotw\alpha)^2}, \\
f_\alpha(l,o,t,w,\alpha) &= \frac{-l^2owt}{(1+lotw\alpha)^2}.
\end{aligned}$$

We see that

$$\begin{aligned}
&f_l(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 1, && f_o(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0, && f_t(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0, \\
&f_w(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0, && f_\alpha(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0.
\end{aligned}$$

The proof now follows by using Theorem 1. □

Theorem 4. *Every positive solution of* (4) *is bounded and* $\lim\limits_{n\to\infty} x_n = 0$.

Proof. It follows from (4) that

$$x_{n+1} = \frac{x_{n-4}}{1 + x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}} \le x_{n-4}.$$

Then the subsequences $\{x_{5n-4}\}_{n=0}^{\infty}$, $\{x_{5n-3}\}_{n=0}^{\infty}$, $\{x_{5n-2}\}_{n=0}^{\infty}$, $\dots$, $\{x_{5n}\}_{n=0}^{\infty}$ are decreasing and so they are bounded from the above by

$$T = \max\{x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0\}.$$

This completes the proof. □

2 Solution of the difference equation $x_{n+1} = \frac{x_{n-4}}{1 - x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}$

Here the specific form of the solutions of the difference equation

$$x_{n+1} = \frac{x_{n-4}}{1 - x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}, \quad n \in \mathbb{N}_0, \tag{5}$$

where the initial conditions are arbitrary nonzero real numbers, will be derived.

Theorem 5. *Let* $\{x_n\}_{n=-4}^{\infty}$ *be a solution of* (5)*. Then for* $n \in \mathbb{N}_0$,

$$x_{5n-4} = \frac{e \prod_{i=0}^{n-1} (1 - 5iabcde)}{\prod_{i=0}^{n-1} (1 - (5i+1)abcde)}, \qquad x_{5n-3} = \frac{d \prod_{i=0}^{n-1} (1 - (5i+1)abcde)}{\prod_{i=0}^{n-1} (1 - (5i+2)abcde)},$$

$$x_{5n-2} = \frac{c \prod_{i=0}^{n-1} (1 - (5i+2)abcde)}{\prod_{i=0}^{n-1} (1 - (5i+3)abcde)}, \qquad x_{5n-1} = \frac{b \prod_{i=0}^{n-1} (1 - (5i+3)abcde)}{\prod_{i=0}^{n-1} (1 - (5i+4)abcde)},$$

$$x_{5n} = \frac{a \prod_{i=0}^{n-1} (1 - (5i+4)abcde)}{\prod_{i=0}^{n-1} (1 - (5i+5)abcde)},$$

where $x_{-4} = e$, $x_{-3} = d$, $x_{-2} = c$, $x_{-1} = b$, $x_0 = a$.

Proof. The proof is similar to the proof of Theorem 2 and therefore it will be omitted. □

Theorem 6. *The equation* (5) *has a unique equilibrium point* $\overline{x} = 0$*, which is not locally asymptotically stable.*

Proof. The proof is the same as the proof of Theorem 3 and hence is omitted. □

3 Solution of the difference equation $x_{n+1} = \frac{x_{n-4}}{-1 + x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}$

In this section, we investigate the solutions of the difference equation

$$x_{n+1} = \frac{x_{n-4}}{-1 + x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}, \quad n \in \mathbb{N}_0, \tag{6}$$

where the initial conditions are arbitrary nonzero real numbers with $x_0 x_{-1} x_{-2} x_{-3} x_{-4} \neq 1$.

Theorem 7. *Let* $\{x_n\}_{n=-4}^{\infty}$ *be a solution of the difference equation* (6). *Then for* $n = 0, 1, 2, \ldots$ *we have*

$$\begin{aligned} x_{10n-9} &= \frac{e}{(abcde-1)}, & x_{10n-8} &= d(abcde-1),\\ x_{10n-7} &= \frac{c}{(abcde-1)}, & x_{10n-6} &= b(abcde-1),\\ x_{10n-5} &= \frac{a}{(abcde-1)}, & x_{10n-4} &= e,\\ x_{10n-3} &= d, & x_{10n-2} &= c,\\ x_{10n-1} &= b, & x_{10n} &= a. \end{aligned}$$

Proof. Suppose that $n > 0$ and that our assumption holds for $n-1$. Then

$$\begin{aligned} x_{10n-19} &= \frac{e}{(abcde-1)}, & x_{10n-18} &= d(abcde-1),\\ x_{10n-17} &= \frac{c}{(abcde-1)}, & x_{10n-16} &= b(abcde-1),\\ x_{10n-15} &= \frac{a}{(abcde-1)}, & x_{10n-14} &= e,\\ x_{10n-13} &= d, & x_{10n-12} &= c,\\ x_{10n-11} &= b, & x_{10n-10} &= a. \end{aligned}$$

Now, it follows from (6) that

$$x_{10n-9} = \frac{x_{10n-14}}{-1 + x_{10n-10}x_{10n-11}x_{10n-12}x_{10n-13}x_{10n-14}} = \frac{e}{-1+abcde}$$

Then, we have

$$x_{10n-9} = \frac{e}{(abcde-1)}.$$

Other relations can be proven similarly. ☐

Theorem 8. *The equation* (6) *has three equilibrium points which are* $0, \pm\sqrt[5]{2}$, *and these equilibrium points are not locally asymptotically stable.*

Proof. The proof is the same as the proof of Theorem 3 and hence is omitted. ☐

Theorem 9. *The equation* (6) *has a periodic solution of period five if* $abcde = 2$, *and then takes the form*

$$\{e, d, c, b, a, \ldots\}.$$

Proof. The proof is obtained from Theorem 7. ☐

4 Solution of the difference equation $x_{n+1} = \frac{x_{n-4}}{-1-x_nx_{n-1}x_{n-2}x_{n-3}x_{n-4}}$

In this section, we investigate the solutions of the difference equation

$$x_{n+1} = \frac{x_{n-4}}{-1 - x_nx_{n-1}x_{n-2}x_{n-3}x_{n-4}}, \quad n \in \mathbb{N}_0, \tag{7}$$

where the initial conditions are arbitrary nonzero real numbers with $x_0x_{-1}x_{-2}x_{-3}x_{-4} \neq -1$.

Theorem 10. *Let $\{x_n\}_{n=-4}^{\infty}$ be a solution of the difference equation* (7)*. Then*

$$\begin{aligned}
x_{10n-9} &= \frac{-e}{(abcde+1)}, & x_{10n-8} &= -d(abcde+1),\\
x_{10n-7} &= \frac{-c}{(abcde+1)}, & x_{10n-6} &= -b(abcde+1),\\
x_{10n-5} &= \frac{-a}{(abcde+1)}, & x_{10n-4} &= e,\\
x_{10n-3} &= d, & x_{10n-2} &= c,\\
x_{10n-1} &= b, & x_{10n} &= a,
\end{aligned}$$

where the initial conditions are arbitrary nonzero real numbers with $x_0x_{-1}x_{-2}x_{-3}x_{-4} \neq -1$.

Proof. The proof is similar to the proof of Theorem 7 and therefore is omitted. □

Theorem 11. *The equation* (7) *has three equilibrium point which are $0, \pm\sqrt[5]{-2}$ and these equilibrium points are not locally asymptotically stable.*

Proof. The proof is the same as the proof of Theorem 3 and hence is omitted. □

Theorem 12. *The equation* (7) *has a periodic solution of period twelve if $abcde = -2$, and then takes the form*

$$\{e, d, c, b, a, \dots\}.$$

Proof. The proof is obtained from Theorem 10. □

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References

[1] Agarwal R.P., Elsayed E.M. *Periodicity and stability of solutions of higher order rational difference equation*. Advanced Studies in Contemporary Mathematics 2008, **17** (2), 181–201.

[2] Aloqeili M. *Dynamics of a rational difference equation*. Appl. Math. Comput. 2006, **176** (2), 768–774. doi: 10.1016/j.amc.2005.10.024

[3] Bilgin A., Kulenović M.R.S. *Global asymptotic stability for discrete single species population models*. Discrete Dyn. Nat. Soc. 2017, **2017**, 1–15. doi:10.1155/2017/5963594

[4] DeVault R., Ladas G., Schultz S.W. *On the recursive sequence $x_{n+1} = \frac{A}{x_n} + \frac{1}{x_{n-2}}$*. Proc. Amer. Math. Soc. 1998, **126** (11), 3257–3261.

[5] Elsayed E.M. *On The Difference Equation $x_{n+1} = \frac{x_{n-5}}{-1+x_{n-2}x_{n-5}}$*. Inter. J. Contemp. Math. Sci. 2008, **33** (3), 1657–1664.

[6] Elsayed E.M. *Behavior of a rational recursive sequences*. Stud. Univ. Babeş-Bolyai Math. 2011, **LVI** (1), 27–42.

[7] Elsayed E.M. *Solution of a Recursive Sequence of Order Ten*. General Mathematics 2011, **19** (1), 145–162.

[8] Elsayed E.M., Alzubaidi M.M. *Expressions and Dynamical Behavior of Rational Recursive Sequences*. Int. J. Comput. Anal. Appl. 2020, **28** (1), 67–79.

[9] Gelişken A. *On A System of Rational Difference Equations*. J. Comput. Anal. Appl. 2017, **23** (4), 593–606.

[10] Karataş R., Cinar C., Şimşek D. *On Positive Solutions of the Difference Equation* $x_{n+1} = \frac{x_{n-5}}{1+x_{n-2}x_{n-5}}$. Int. Journal of Contemp. Math. Sciences 2006, **10** (1), 495–500.

[11] Kocic V.L., Ladas G. Global behavior of nonlinear difference equations of higher order with applications. In: (MAIA) Mathematics and its Applications, 256. Springer, Dordrecht, 1993. doi:10.1007/978-94-017-1703-8

[12] Kulenovic M.R.S., Ladas G. Dynamics of second order rational difference equations. With open problems and conjectures. Chapman & Hall/CRC, New York, 2001. doi:10.1201/9781420035384

[13] Şimşek D., Cinar C., Yalçinkaya I. *On the recursive sequence* $x_{n+1} = \frac{x_{n-3}}{1+x_{n-1}}$. Int. Journal of Contemp. Math. Sciences 2006, **9** (12), 475–480. doi:10.12988/ijcms.2006.06052

[14] Şimşek D., Cinar C., Yalçinkaya I. *On the recursive sequence* $x_{n+1} = \frac{x_{n-5}}{1+x_{n-2}}$. Int J. Pure Appl. Math. 2006, **27**, 501–507.

[15] Şimşek D., Cinar C., Yalçinkaya I. *On the recursive sequence* $x_{n+1} = \frac{x_{n-5}}{1+x_{n-1}x_{n-3}}$. Int J. Pure Appl. Math. 2006, **28**, 117–124.

[16] Şimşek D., Abdullayev F. *On the recursive sequence* $x_{n+1} = \frac{x_{n-(4k+3)}}{1+\prod_{t=0}^{2} x_{n-(k+1)t-k}}$. J. Math. Sci. 2017, **222** (6), 762–771.

[17] Tasdemir E. *On the global asymptotic stability of a system of difference equations with quadratic terms*. J. Appl. Math. Comput. 2020, 1–15.

[18] Tollu D.T., Yalçinkaya I. *Global behavior of a three-dimensional system of difference equations of order three*. Com mun. Fac. Sci. Univ. Ank. Ser. A1 M ath. Stat. 2019, **68** (1), 1–16. doi:10.31801/cfsuasm as.443530

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У цій статті ми вивчаємо якісну поведінку раціонального різницевого рівняння

$$x_{n+1} = \frac{x_{n-4}}{\pm 1 \pm x_n x_{n-1} x_{n-2} x_{n-3} x_{n-4}}, \quad n \in \mathbb{N}_0 := \{0\} \cup \mathbb{N},$$

з довільними ненульовими дійсними початковими умовами. Основна мета цієї статті — отримати форми розв'язків нелінійних різницевих рівнянь п'ятого порядку з довільними додатними дійсними початковими умовами. Більше того, ми досліджуємо стійкість, обмеженість, коливний та періодичний характер цих розв'язків. Результати цієї статті покращують та розширюють деякі відповідні результати в літературі.

Ключові слова і фрази: різницеве рівняння, рекурсивна послідовність, періодичний розв'язок.