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OPERATOR CALCULUS ON THE CLASS OF SATO'S HYPERFUNCTIONS

We construct a functional calculus for generators of analytic semigroups of operators on a Banach space. The symbol class of the calculus consists of hyperfunctions with a compact support in $[0, \infty)$. Domain of constructed calculus is dense in the Banach space.

Key words and phrases: functional calculus, analytic semigroups of operators, hyperfunctions.

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INTRODUCTION

Roughly speaking, the aim of functional calculus is to define an operator $f(A)$ for a function f belonging to some algebra of functions (so called symbol algebra) and for some (in general unbounded) operator A on a Banach space. In the same time we understand a functional calculus as an algebraic (or more generally topological) isomorphism from symbol algebra to algebra of operators.

There are many ways to define a functional calculus for different classes of operators on different symbol algebras. One of them (the Hille-Phillips calculus) was developed in [6] and generalized in [10, 2, 9, 8]. For new helpful applications of a Hille-Phillips type functional calculus see [1] and the references given there.

In this article we use the class of hyperfunctions, supported by a compact set in positive semiaxis, as a symbol algebra and construct an analogue of Hille-Phillips calculus for generators of analytic semigroups of operators on a Banach space.

The hyperfunctions were introduced by M.Sato in [11]. We can understand Sato's hyperfunctions as a generalization of the concept of boundary values of complex analytic functions and as an extension of ultradistributions with a compact support [7]. Theory of hyperfunctions is a very useful tool in the study of D-modules, holonomic systems of differential equations, and especially some aspects of symplectic geometry and harmonic analysis that are part of microlocal analysis, especially algebraic microlocalization.

1 PRELIMINARIES AND DENOTATIONS

Let $\mathscr{L}(X)$ denote the space of continuous linear operators over a locally convex space X and let X' be the dual of X. Throughout the paper, the spaces $\mathscr{L}(X)$ and X' will be endowed with the locally convex topology of uniform convergence on bounded subsets of X.

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Let $\mathbb{R}_+ := [0,\infty)$ stand for the nonnegative semiaxis. A family $\{U(t) : t \in \mathbb{R}_+\}$ of bounded linear operators on a complex Banach space $(E, \|\cdot\|)$ is called a one-parameter semigroup if $U(\cdot)$ is a mapping $U(\cdot)\colon \mathbb{R}_+ \longmapsto \mathscr{L}(E)$ such that $U(t+s) = U(t)U(s)$ and $U(0) = I$ is the unit operator. The operator

$$Ax := \lim_{t\to+0} \frac{U(t)x - x}{t}, \qquad x \in \mathfrak{D}(A),$$

where $\mathfrak{D}(A)$ consists of all $x \in E$ for which the previous limit exists, is called a generator of the semigroup $\{U(t) : t \in \mathbb{R}_+\}$. To emphasize that an operator A generates a semigroup, we will use the standard notation $\{e^{tA} : t \in \mathbb{R}_+\}$ or $\{e^{tA}\}_{t\in\mathbb{R}_+}$ instead of $\{U(t) : t \in \mathbb{R}_+\}$.

The semigroup $\{e^{tA} : t \in \mathbb{R}_+\}$ is a C_0-semigroup iff $\lim_{t\to+0} \|e^{tA}x - x\| = 0$ for all $x \in E$. If $\{e^{tA}\}_{t\in\mathbb{R}_+}$ is a C_0-semigroup then the following properties hold (see [3]):

- if $x \in \mathfrak{D}(A)$ then $e^{tA}x \in \mathfrak{D}(A)$ and $Ae^{tA}x = e^{tA}Ax$,
- $e^{tA}x \in \mathfrak{D}(A)$ for all $x \in E$, $t \in \mathbb{R}_+$ and $\mathfrak{D}(A)$ is dense in E.

Let Σ_θ be an open sector in $\mathbb{C}$, defined as

$$\Sigma_\theta := \{z \in \mathbb{C} : |\arg z| < \theta\}\backslash\{0\}.$$

It is obvious that closure of a sector Σ_θ is defined as $\Sigma_\theta^{\mathrm{cl}} := \{z \in \mathbb{C} : |\arg z| \le \theta\}$.

We say that a bounded C_0-semigroup $\{U(t) : t \in \mathbb{R}_+\}$ on a Banach space E is a bounded analytic semigroup (see [5, 12]), if there exists $0 < \theta \le \frac{\pi}{2}$ such that

- $U(t)$ is a restriction onto $\mathbb{R}_+$ of an analytic family of operators $U(z)$ in open sector Σ_θ;
- $U(s+z) = U(s) \circ U(z)$ for all $s, z \in \Sigma_\theta^{\mathrm{cl}}$;
- for each $\vartheta < \theta$ the family $\{U(z)\}$ is uniformly bounded in $\Sigma_\vartheta^{\mathrm{cl}}$ and $U(z)x \to x$ as $z \to 0$ in $\Sigma_\vartheta^{\mathrm{cl}}$ for each $x \in E$.

Let $H(W)$ denote a vector space of all holomorphic functions on an open set $W \subset \mathbb{C}$. We follow [4] in defining the space of functions

$$H := \lim_K \operatorname{ind}(\lim_k \operatorname{pr} H_{K,k}),$$

where

$$H_{K,k} := \Big\{F \in H(\Omega_K) : \|F\|_{K,k} := \sup_{z\in\Omega_K} |F(z)|e^{k\mathrm{Re} z} < \infty\Big\},$$

and

$$\Omega_K := \left\{z \in \mathbb{C} : |\mathrm{Im} z| < \frac{\mathrm{Re} z}{K} + \frac{1}{K^2}\right\}. \tag{1}$$

In other words, H is the space of functions F, which are holomorphic in a some angular neighborhood Ω_K of $[0,\infty)$ with finite norms $\|F\|_{K,k}$ for each k.

Here and subsequently, $\mathcal{A}(\Omega)$ denotes the space of real-analytic functions in an open set $\Omega \in \mathbb{R}$. Let $\mathcal{A}(\mathbb{R}_+)$ denote the space of germs of real-analytic functions on neighborhoods of the semiaxis $[0,\infty)$. A restriction of any element of $\mathcal{A}(\mathbb{R}_+)$ is uniquely defined function on $\mathbb{R}_+$. In the sequel we will treat $\mathcal{A}(\mathbb{R}_+)$ as the space of such restrictions.

It is clear, that restrictions of functions from H onto $[0,\infty)$ form a subset in $\mathcal{A}(\mathbb{R}_+)$, which we will denote by the symbol H_+. Since $\mathbb{R}_+ \subset \Omega_K$, for any bounded function $f(z)$ the inequality $\sup_{z\in\mathbb{R}_+}|f(z)| \le \sup_{z\in\Omega_K}|f(z)|$ holds. Therefore, $\varphi \in H_+$ iff it is a real-analytic function on $\mathbb{R}_+$ satisfying the condition

$$\sup_{t\in\mathbb{R}_+}|\varphi(t)|e^{kt} < \infty$$

for each k and it can be continued in an angle Ω_K for some K.

Let Ω be an open set in $\mathbb{R}$ and V be an open set in $\mathbb{C}$ containing Ω as a relatively closed set. The vector space of all hyperfunctions on Ω is defined (see [7, 11]) to be the quotient space

$$\mathcal{B}(\Omega) = H(V\setminus\Omega)/H(V),$$

where $H(V)$ denotes the restriction of $H(V)$ to $V\setminus\Omega$. The hyperfunction represented by an $F\in H(V\setminus\Omega)$ is denoted as follows

$$f = [F] = F(t+i0) - F(t-i0) \quad \text{or} \quad f(t) = [F(z)]_{z=t}.$$

The representative F is called a defining function of the hyperfunction f.

The set of all hyperfunctions with a support in a fixed compact set $K\subset\Omega$ is represented as $\mathcal{B}_K(\Omega) = H(V\setminus K)/H(V)$. Let $\mathcal{B}_c(\Omega)$ denote the space of hyperfunctions with a compact support in Ω.

The following statement from [7] will be used in the sequel.

Theorem 1. *Let $\Omega\subset\mathbb{R}$ be an open set. Then we have the isomorphism of vector spaces $\mathcal{B}_c(\Omega)\cong\mathcal{A}(\Omega)'$. For a $\varphi\in\mathcal{A}(\Omega)$ and an $f=[F]\in\mathcal{B}_c(\Omega)$ with $F\in H(V\setminus\operatorname{supp} f)$, the canonical bilinear functional is given by*

$$\langle f,\varphi\rangle = -\oint_\Gamma F(z)\boldsymbol{\varphi}(z)dz, \tag{2}$$

where Γ is a closed path in the intersection of the domain of the analytic continuation $\boldsymbol{\varphi}$ of a function φ and the domain of F, and surrounding $\operatorname{supp} f$ once in the positive orientation.

2 CROSS-CORRELATION

Let us denote by $\mathcal{B}_c(\mathbb{R}_+)$ the space of all hyperfunctions with a compact support in the semiaxis $[0,\infty)$. For any $f=[F]$ and $g=[G]$ from $\mathcal{B}_c(\mathbb{R}_+)$ we define the convolution $f*g$ by $f*g=[H]$, where

$$H(z) = -\oint_\Gamma F(w)G(z-w)dw,$$

and Γ is a closed path in the intersection of the domains of analytic functions $w\longmapsto F(w)$ and $w\longmapsto G(z-w)$. It is known [7], that the space $\mathcal{B}_c(\mathbb{R}_+)$ is an algebra with respect to the convolution with Dirac delta-function $\delta(x)$ as an unit element.

The cross-correlation of a hyperfunction $f=[F]\in\mathcal{B}_c(\mathbb{R}_+)$ and a real-analytic function $\varphi\in\mathcal{A}(\mathbb{R}_+)$ is defined to be

$$(f\star\varphi)(t) := -\oint_\Gamma F(z)\boldsymbol{\varphi}(z+t)dz, \qquad t\in\mathbb{R}_+,$$

where Γ is a closed path in the intersection of a domain of the analytic function $z\longmapsto F(z)$ and a domain of $z\longmapsto\boldsymbol{\varphi}(z+t)$ (here $\boldsymbol{\varphi}$ is the analytic continuation of φ), and surrounding $\operatorname{supp} f$ once in the positive orientation. The correctness of the definition follows from Theorem 1.

Theorem 2. *For a hyperfunction $f = [F] \in \mathcal{B}_c(\mathbb{R}_+)$ and a function $\varphi \in \mathcal{A}(\mathbb{R}_+)$ the cross-correlation $f \star \varphi$ is a real-analytic function, belonging to $\mathcal{A}(\mathbb{R}_+)$.*

Proof. According to Pringsheim's theorem [7, Theorem 2.1], an infinitely differentiable function φ belongs to $\mathcal{A}(\mathbb{R}_+)$ iff for each compact set $K \subset \mathbb{R}_+$ there exist constants $h > 0$ and $C > 0$ such that inequality

$$\sup_{x\in K} |\varphi^{(n)}(x)| \le Ch^n n!$$

holds for any $n \in \mathbb{Z}_+$.

Let $K \subset \mathbb{R}_+$ be a compact set. The following inequalities hold

$$\begin{aligned}\sup_{t\in K} |(f\star\varphi)^{(n)}(t)| &= \sup_{t\in K} \Big| -\oint_\Gamma F(z)\boldsymbol{\varphi}^{(n)}(z+t)dz\Big| \le \sup_{t\in K} \oint_\Gamma |F(z)|\cdot|\boldsymbol{\varphi}^{(n)}(z+t)|\,dz \\ &\le \sup_{t\in K}\sup_{z\in\Gamma} |\boldsymbol{\varphi}^{(n)}(z+t)| \oint_\Gamma |F(z)|\,dz \le \sup_{t\in K}\sup_{z\in\Gamma} |\boldsymbol{\varphi}^{(n)}(z+t)| \sup_{z\in\Gamma}|F(z)|\mu(\Gamma),\end{aligned} \tag{3}$$

where $\mu(\Gamma)$ denotes the length of Γ. Note, that $\sup_{z\in\Gamma}|F(z)|\mu(\Gamma) < \infty$.

Via the maximum-modulus principle in complex analysis there exists a point $z_0 \in \Gamma$, such that $\sup_{t\in K}\sup_{z\in\Gamma}|\boldsymbol{\varphi}^{(n)}(z+t)| = \sup_{t\in K}|\boldsymbol{\varphi}^{(n)}(z_0+t)|$. The function $\mathbb{R} \ni t \longmapsto |\boldsymbol{\varphi}^{(n)}(z_0+t)|$ is a restriction of the analytic function $\mathbb{C} \ni z \longmapsto |\boldsymbol{\varphi}^{(n)}(z_0+z)|$, therefore $|\boldsymbol{\varphi}^{(n)}(z_0+t)|$ is a real-analytic function. So, by Pringsheim's theorem there exist constants $h > 0$ and $C > 0$ such that $\sup_{t\in K}|\boldsymbol{\varphi}^{(n)}(z_0+t)| \le Ch^n n!$.

Finally, we can continue the inequality (3) as follows

$$\sup_{t\in K}|(f\star\varphi)^{(n)}(t)| \le \mu(\Gamma)\sup_{z\in\Gamma}|F(z)|Ch^n n! = C_1 h^n n!\,,$$

where $C_1 := \mu(\Gamma)\sup_{z\in\Gamma}|F(z)|C$, which proves that $(f\star\varphi)(t) \in \mathcal{A}(\mathbb{R}_+)$. $\square$

The following statement may be considered as an improvement of Theorem 2.

Theorem 3. *For a hyperfunction $f = [F] \in \mathcal{B}_c(\mathbb{R}_+)$ and a function $\varphi \in H_+$ the cross-correlation $f \star \varphi$ belongs to H_+.*

Proof. Note, that function $t \longmapsto (f\star\varphi)(t)$ can be continued to the analytic one

$$\Omega_K \ni s \longmapsto (f\star\varphi)(s) := -\oint_\Gamma F(z)\boldsymbol{\varphi}(z+s)dz,$$

for some K, since $\boldsymbol{\varphi} \in H$.

Since bilinear form (2) does not depend on choice of the path Γ, we assume, that a domain (1) always contain the path. Then for each k we have

$$\begin{aligned}\sup_{t\in\mathbb{R}_+}|(f\star\varphi)(t)|\,e^{kt} &= \sup_{t\in\mathbb{R}_+}\Big|-\oint_\Gamma F(z)\boldsymbol{\varphi}(z+t)\,dz\Big|\,e^{kt} \le \sup_{s\in\Omega_K}\Big|-\oint_\Gamma F(z)\boldsymbol{\varphi}(z+s)e^{ks}\,dz\Big| \\ &= \sup_{s\in\Omega_K}\Big|-\oint_\Gamma F(z)\boldsymbol{\varphi}(z+s)e^{k(s+z)}e^{-kz}\,dz\Big| \le \sup_{s\in\Omega_K}\oint_\Gamma |F(z)||\boldsymbol{\varphi}(z+s)|e^{k\mathrm{Re}(s+z)}e^{-k\mathrm{Re}z}\,dz \\ &\le \sup_{z\in\Gamma}\sup_{s\in\Omega_K}|\boldsymbol{\varphi}(z+s)|e^{k\mathrm{Re}(s+z)}\oint_\Gamma |F(z)|e^{-k\mathrm{Re}z}\,dz.\end{aligned} \tag{4}$$

In the case $0 \notin \operatorname{supp} f$ it is possible to choose the curve Γ such that $\Gamma \subset V_K$, where $V_K := \left\{z \in \mathbb{C} : |\mathrm{Im} z| < \frac{\mathrm{Re} z}{K}\right\}$. Note, that inequalities

$$\left|\mathrm{Im}(s+z)\right| = \left|\mathrm{Im} s + \mathrm{Im} z\right| \leq \left|\mathrm{Im} s\right| + \left|\mathrm{Im} z\right| < \frac{\mathrm{Re} s}{K} + \frac{1}{K^2} + \frac{\mathrm{Re} z}{K} = \frac{\mathrm{Re}(s+z)}{K} + \frac{1}{K^2}$$

imply $r = s + z \in \Omega_K$ for any $s \in \Omega_K$ and $z \in V_K$. Since $\boldsymbol{\varphi} \in H$, the inequality (4) can be continued as follows

$$\begin{aligned}\sup_{t \in \mathbb{R}_+} |(f \star \varphi)(t)|\, e^{kt} &\leq \sup_{z \in \Gamma} \sup_{s \in \Omega_K} |\boldsymbol{\varphi}(z+s)| e^{k\mathrm{Re}(s+z)} \oint_\Gamma |F(z)| e^{-k\mathrm{Re} z}\, dz \\ &= \sup_{r \in \Omega_K} |\boldsymbol{\varphi}(r)| e^{k\mathrm{Re} r} \oint_\Gamma |F(z)| e^{-k\mathrm{Re} z}\, dz < \infty.\end{aligned}$$

Consider the case $0 \in \operatorname{supp} f$. Let us use Ω_{2K} instead of Ω_K in the estimation (4). Then we obtain

$$\sup_{t \in \mathbb{R}_+} |(f \star \varphi)(t)|\, e^{kt} \leq \sup_{z \in \Gamma} \sup_{s \in \Omega_{2K}} |\boldsymbol{\varphi}(z+s)| e^{k\mathrm{Re}(s+z)} \oint_\Gamma |F(z)| e^{-k\mathrm{Re} z}\, dz. \tag{5}$$

Inequalities

$$\left|\mathrm{Im}(s+z)\right| = \left|\mathrm{Im} s + \mathrm{Im} z\right| \leq \left|\mathrm{Im} s\right| + \left|\mathrm{Im} z\right| < \frac{\mathrm{Re} s}{2K} + \frac{1}{4K^2} + \frac{\mathrm{Re} z}{2K} + \frac{1}{4K^2} < \frac{\mathrm{Re}(s+z)}{K} + \frac{1}{K^2}$$

imply $r = z + s \in \Omega_K$ for any $z \in \Omega_{2K}$ and $s \in \Omega_{2K}$. Since $\Gamma \subset \Omega_{2K}$ and $\boldsymbol{\varphi} \in H$, the inequality (5) can be continued as follows

$$\begin{aligned}\sup_{t \in \mathbb{R}_+} |(f \star \varphi)(t)|\, e^{kt} &\leq \sup_{z \in \Gamma} \sup_{s \in \Omega_{2K}} |\boldsymbol{\varphi}(z+s)| e^{k\mathrm{Re}(s+z)} \oint_\Gamma |F(z)| e^{-k\mathrm{Re} z}\, dz \\ &= \sup_{r \in \Omega_K} |\boldsymbol{\varphi}(r)| e^{k\mathrm{Re} r} \oint_\Gamma |F(z)| e^{-k\mathrm{Re} z}\, dz < \infty.\end{aligned}$$

Hence, $f \star \varphi \in H_+$. □

3 Operator calculus

Let A be a generator of an analytic semigroup $\{e^{tA}\}_{t \in \mathbb{R}_+}$. Let $D_+(A)$ be a subspace in the Banach space E, defined by

$$D_+(A) := \left\{ \widehat{x}_{(A)} : \widehat{x}_{(A)} := \int_0^{+\infty} e^{tA} x \varphi(t) dt, \quad x \in E, \varphi \in H_+ \right\}.$$

Note, that we understand the above integral in the Bochner sense [6].

Theorem 4. *The subspace $D_+(A)$ is dense in E.*

Proof. Suppose that $D_+(A)$ is not dense in E. Then by Hahn-Banach's theorem there is nonzero functional $x' \in E'$ such that $\langle x', \widehat{x}_{(A)} \rangle = 0$ for all $x \in \mathfrak{D}(A^\infty)$, where $\mathfrak{D}(A^\infty) := \bigcap_{\alpha \in \mathbb{Z}_+} \mathfrak{D}(A^\alpha)$, and $\mathfrak{D}(A^\alpha)$ is the domain of operator A^α.

From the Bochner's integral properties [6, 3.7] it follows that

$$\langle x', \widehat{x}_{(A)} \rangle = \int_0^{+\infty} \langle x', e^{tA} x \rangle \varphi(t) dt = 0.$$

Since A is a generator of a C_0-semigroup, $\mathfrak{D}(A^\infty)$ is dense in E (see [5]). It follows that for any $x \in \mathfrak{D}(A^\infty)$ the real-analytic function $t \longmapsto \langle x', e^{tA}x \rangle$ must vanish identically on $[0, +\infty)$ since otherwise it would have been possible to choose $\varphi \in H_+$ such that $\langle x', \widehat{x}_{(A)} \rangle$ does not vanish. Thus in particular for $t = 0$ we obtain that equality $\langle x', x \rangle = 0$ holds for every $x \in \mathfrak{D}(A^\infty)$. Therefore $x' = 0$ which contradicts the choice of x'. $\square$

For each hyperfunction $f = [F] \in \mathcal{B}_c(\mathbb{R}_+)$ the operator $f(A)$ is given by

$$f(A) : D_+(A) \ni \widehat{x}_{(A)} \longmapsto f(A)\widehat{x}_{(A)} = \int_0^{+\infty} e^{tA} x (f \star \varphi)(t) dt \in D_+(A).$$

The mapping $\Phi_A : B_c(\mathbb{R}_+) \ni f \longmapsto f(A) \in \mathscr{L}(D_+(A))$ is called the operator calculus for generators of analytic semigroups of operators on the class of Sato's hyperfunctions with a compact support in $\mathbb{R}_+$.

Theorem 5. *For each $\varphi \in H_+$ the mapping Φ_A produces an algebraic homomorphism from the convolution algebra $B_c(\mathbb{R}_+)$ into the algebra of linear continuous operators over $D_+(A)$.*

Proof. Let $f, g \in B_c(\mathbb{R}_+)$, $\alpha, \beta \in \mathbb{R}$. The linearity $\Phi_A(\alpha f + \beta g) = \alpha \Phi_A(f) + \beta \Phi_A(g)$ of the mapping Φ_A is clear via the linearity of the integral.

Let us show that $\Phi_A(f * g) = \Phi_A(f) \circ \Phi_A(g)$. Let F and G be defining functions of the hyperfunctions f and g respectively. Using the method of variable changing, for an arbitrary $\widehat{x}_{(A)} \in D_+(A)$ we obtain

$$\begin{aligned}
(f * g)(A)\widehat{x}_{(A)} &= \int_0^{+\infty} e^{tA} x ((f * g) \star \varphi)(t)\, dt \\
&= \int_0^{+\infty} e^{tA} x \Big(- \oint_{\Gamma_1} \Big(- \oint_{\Gamma_2} F(w) G(z - w)\, dw \Big) \boldsymbol{\varphi}(z + t)\, dz \Big)\, dt \\
&= \int_0^{+\infty} e^{tA} x \Big(- \oint_{\Gamma_2} F(w) \Big(- \oint_{\Gamma_1} G(z - w) \boldsymbol{\varphi}(z + t)\, dz \Big)\, dw \Big)\, dt \\
&= \int_0^{+\infty} e^{tA} x \Big(- \oint_{\Gamma_2} F(w) \Big(- \oint_{\Gamma_3} G(z) \boldsymbol{\varphi}(z + w + t)\, dz \Big)\, dw \Big)\, dt \\
&= \int_0^{+\infty} e^{tA} x (f \star (g \star \varphi))(t)\, dt = f(A) \int_0^{+\infty} e^{tA} x (g \star \varphi)(t) dt = f(A) g(A) \widehat{x}_{(A)},
\end{aligned}$$

where Γ_1, Γ_2 are suitable curves and Γ_3 is a linear shift of the Γ_1.

For Dirac delta-function $\delta(x)$ we have

$$\delta(A)\widehat{x}_{(A)} = \int_0^{+\infty} e^{tA} x (\delta \star \varphi)(t) dt = \int_0^{+\infty} e^{tA} x \varphi(t) dt = \widehat{x}_{(A)}.$$

So, $\delta(A)$ is an unit operator on $D_+(A)$. $\square$

REFERENCES

[1] Baeumer B., Haase M., Kovács M. *Unbounded functional calculus for bounded groups with applications*. J. Evol. Eqv. 2009, **9** (1), 171–195. doi:10.1007/s00028-009-0012-z

[2] Balakrishnan A.V. *An operational calculus for infinitesimal operators of semigroups*. Trans. Amer. Math. Soc. 1960, **91**, 330–353. doi: 10.1090/S0002-9947-1959-0107179-0

[3] Butzer P.L., Berens H. Semi-groups of operators and approximation. Springer, London, 2012.

[4] Domanski P., Langenbruch M. *On the Laplace Transform for Vector Valued Hyperfunctions.* Functiones et Approximatio 2010, **43** (2), 129–159. doi:10.7169/facm/1291903394

[5] Engel K.-J., Nagel R. One-parameter semigroups for linear evolution equations. Graduate Text in Mathematics, **194**. Springer-Verlag, Berlin, 2000.

[6] Hille E., Phillips R. Functional Analysis and Semi-Groups. Amer. Math. Soc., Coll. Publ., **31**, Providence R.I., 2000.

[7] Komatsu H. An Introduction to the Theory of Generalized Functions. University Publ., Tokyo, 2000.

[8] Lopushansky O.V., Sharyn S.V. *Functional calculus for generators of analytic semigroups of operators.* Carpath. Math. Publ. 2012, **4** (1), 83–89. (in Ukrainian)

[9] Lopushansky O.V., Sharyn S.V. *Operator calculus for convolution algebra of Schwartz distributions on semiaxis.* Mat. Stud. 1997, **7** (1), 61–72.

[10] Nelson E.A. *Functional Calculus Using Singular Laplace Integrals*. Trans. Amer. Math. Soc. 1958, **88**, 400–413. doi: 10.1090/S0002-9947-1958-0096136-8

[11] Sato M. *Theory of hyperfunctions. I, II.* J. Fac. Sci. Univ. Tokyo 1959/60, **8**, 139–193, 387–436.

[12] Vrabie I.I. C_0-semigroups and Applications. Mathematics Studies, **191**. North-Holland, Amsterdam, 2003.

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Патра М.І., Шарин С.В. *Функціональне числення в класі гіперфункцій Сато* // Карпатські математичні публікації. — 2013. — Т.5, №1. — С. 114–120.

Ми будуємо функціональне числення для генераторів аналітичних напівгруп операторів на банаховому просторі. Клас символів такого числення складається з гіперфункцій з компактними носіями в $[0,\infty)$. Область визначення побудованого числення є щільною в банаховому просторі.

Ключові слова і фрази: функціональне числення, аналітичні напівгрупи операторів, гіперфункції.

Патра М.И., Шарин С.В. *Функциональное исчисление в классе гиперфункций Сато* // Карпатские математические публикации. — 2013. — Т.5, №1. — С. 114–120.

Ми строим функциональное исчисление для генераторов аналитических полугрупп операторов, действующих в банаховом пространстве. Класс символов такого исчисления состоит из гиперфункций с компактными носителями в $[0,\infty)$. Область определения построенного исчисления плотна в банаховом пространстве.

Ключевые слова и фразы: функциональное исчисление, аналитические полугруппы операторов, гиперфункции.