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ON SOME PERTURBATIONS OF A STABLE PROCESS AND SOLUTIONS OF THE CAUCHY PROBLEM FOR A CLASS OF PSEUDO- DIFFERENTIAL EQUATIONS

A fundamental solution of some class of pseudo-differential equations is constructed by a method based on the theory of perturbations. We consider a symmetric α-stable process in multidimensional Euclidean space. Its generator $\mathbf{A}$ is a pseudo-differential operator whose symbol is given by $-c|\lambda|^{\alpha}$, where the constants $\alpha \in (1,2)$ and $c > 0$ are fixed. The vector-valued operator $\mathbf{B}$ has the symbol $2ic|\lambda|^{\alpha-2}\lambda$. We construct a fundamental solution of the equation $u_t = (\mathbf{A} + (a(\cdot), \mathbf{B}))u$ with a continuous bounded vector-valued function a.

Key words and phrases: stable process, Cauchy problem, pseudo-differential equation, transition probability density.

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INTRODUCTION

Let $\mathbf{A}$ denote a pseudo-differential operator that acts on a twice continuously differentiable bounded function $(\varphi(x))_{x\in\mathbb{R}^d}$ according to the following rule

$$(\mathbf{A}\varphi)(x) = \frac{c}{\varkappa}\int_{\mathbb{R}^d} \frac{\varphi(x+y) - \varphi(x) - (y, \nabla\varphi(x))}{|y|^{d+\alpha}}\, dy, \tag{1}$$

where $c > 0$, $1 < \alpha < 2$, $d \in \mathbb{N}$ are some constants, $\varkappa = -\dfrac{2\pi^{\frac{d-1}{2}}\Gamma(2-\alpha)\Gamma\left(\frac{\alpha+1}{2}\right)\cos\frac{\pi\alpha}{2}}{\alpha(\alpha-1)\Gamma\left(\frac{d+\alpha}{2}\right)}$ and ∇ is the Hamilton operator (gradient). Here $(\cdot,\cdot)$ denotes the scalar product in $\mathbb{R}^d$.

It is known that the function $u(t,x) = \int_{\mathbb{R}^d} \varphi(y) g(t,x,y)\, dy$, where

$$g(t,x,y) = \frac{1}{(2\pi)^d}\int_{\mathbb{R}^d} e^{i(y-x,\lambda)-ct|\lambda|^{\alpha}}\, d\lambda, \tag{2}$$

is a solution of the following Cauchy problem

$$\begin{aligned} \frac{\partial u(t,x)}{\partial t} &= \mathbf{A}_x u(t,x), \quad t > 0,\ x \in \mathbb{R}^d, \\ u(0+,x) &= \varphi(x), \quad x \in \mathbb{R}^d, \end{aligned} \tag{3}$$

for any bounded continuous function $(\varphi(x))_{x\in\mathbb{R}^d}$.

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If an operator acts on a function of several arguments, then it will be provided by a corresponding subscript, for example, $\mathbf{A}_x$ in (3) means that the operator $\mathbf{A}$ is acting on $u(t,x)$ as the function of the variable x.

Note, that the function $(g(t,x,y))_{t>0,x\in\mathbb{R}^d,y\in\mathbb{R}^d}$ serves as transition probability density of a Markov process in $\mathbb{R}^d$, called a symmetric stable process. The operator $\mathbf{A}$ is the generator of it.

Let us consider the equation

$$\frac{\partial u(t,x)}{\partial t} = \mathbf{A}_x u(t,x) + (a(x), \mathbf{B}_x u(t,x)), \quad t>0,\ x\in\mathbb{R}^d, \tag{4}$$

with some $\mathbb{R}^d$-valued function $(a(x))_{x\in\mathbb{R}^d}$ and d-dimensional pseudo-differential operator $\mathbf{B}$ of the order less than α.

In this article, we consider the case, where the a is a bounded continuous function and the operator $\mathbf{B}$ is defined on a differentiable bounded function $(\varphi(x))_{x\in\mathbb{R}^d}$ by the equality

$$(\mathbf{B}\varphi)(x) = \frac{2c}{\alpha\varkappa}\int_{\mathbb{R}^d} \frac{\varphi(x+y)-\varphi(x)}{|y|^{d+\alpha}}\, y\, dy.$$

Note, that $\mathbf{A} = \frac{1}{2}\mathbf{div}(\mathbf{B})$.

We construct a fundamental solution of equation (4) by perturbing the transition probability density of a symmetric stable process. The fundamental solution of equation (4) was constructing in [2] under the assumption that the function a satisfied Holder's condition.

Symmetric stable processes were perturbed by terms of the type $(a(x),\nabla)$ under various assumptions on the function $(a(x))_{x\in\mathbb{R}^d}$ in many papers (see, for example, [1, 3, 5, 6]). The perturbation of stable processes with delta-function in coefficient is constructed in [4].

1 Perturbation of a stable process

We consider a function $(G(t,x,y))_{t>0,x\in\mathbb{R}^d,y\in\mathbb{R}^d}$ as a result of perturbing the transition probability density $g(t,x,y)$ of a symmetric stable process, if it is a solution of the following equation

$$G(t,x,y) = g(t,x,y) + \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,z)(\mathbf{B}_z G(\tau,z,y), a(z))\, dz. \tag{5}$$

Now we define a function $(e(x))_{x\in\mathbb{R}^d}$ by the equality $e(x) = \frac{1}{|a(x)|}a(x)$ for $x\in\mathbb{R}^d$ such that $|a(x)|\neq 0$ and an arbitrary value (with preservation of the measurability) otherwise. Then the equation (5) takes the form

$$G(t,x,y) = g(t,x,y) + \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,z)(\mathbf{B}_z G(\tau,z,y), e(z))|a(z)|\, dz. \tag{6}$$

It is easy to establish the following equality using the representation (2) and integration by parts $\mathbf{B}_x g(t,x,y) = \frac{2}{\alpha}\frac{y-x}{t}g(t,x,y)$. Denote by $V_0(t,x,y)$ a function that is given by the equality

$$V_0(t,x,y) = (\mathbf{B}_x g(t,x,y), e(x)) = \frac{2}{\alpha}\frac{(y-x,e(x))}{t}g(t,x,y). \tag{7}$$

We will construct the solution of (6) in the form

$$G(t,x,y) = g(t,x,y) + \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,z)V(\tau,z,y)|a(z)|\, dz, \tag{8}$$

where the function $V(t,x,y)$ satisfies the equation

$$V(t,x,y) = V_0(t,x,y) + \int_0^t d\tau \int_{\mathbb{R}^d} V_0(t-\tau,x,z)V(\tau,z,y)|a(z)|\,dz. \tag{9}$$

The equation (9) can be solved by the method of successive approximations, namely its solution will be found in the form

$$V(t,x,y) = \sum_{k=0}^{\infty} V_k(t,x,y), \tag{10}$$

where $V_0(t,x,y)$ is defined by the equality (7) and for $k \geq 1$ the following equality

$$V_k(t,x,y) = \int_0^t d\tau \int_{\mathbb{R}^d} V_0(t-\tau,x,z)V_{k-1}(\tau,z,y)|a(z)|\,dz$$

is valid.

The well-known estimate (see [2]) ($t > 0$, $x \in \mathbb{R}^d$, $y \in \mathbb{R}^d$, and $N > 0$ is a constant)

$$g(t,x,y) \leq N\frac{t}{(t^{1/\alpha} + |y-x|)^{d+\alpha}} \tag{11}$$

allows us to write down

$$|V_0(t,x,y)| \leq \frac{2}{\alpha}N\frac{|y-x|}{(t^{1/\alpha} + |y-x|)^{d+\alpha}} \leq \frac{2}{\alpha}\frac{N}{(t^{1/\alpha} + |y-x|)^{d+\alpha-1}}.$$

Then, we get that the inequality

$$|V_k(t,x,y)| \leq \|a\|\frac{2N}{\alpha}\int_0^t d\tau \int_{\mathbb{R}^d} \frac{1}{((t-\tau)^{1/\alpha} + |z-x|)^{d+\alpha-1}}|V_{k-1}(\tau,z,y)|\,dz$$

is true, where $\|a\| = \sup_{x\in\mathbb{R}^d} |a(x)|$.

In order to estimate V_k we make use of the following inequality (see [2])

$$\int_0^t d\tau \int_{\mathbb{R}^d} \frac{1}{((t-\tau)^{1/\alpha} + |z-x|)^{d+\alpha-1}} \cdot \frac{\tau^\delta}{(\tau^{1/\alpha} + |z-x|)^{d+\alpha-1}}\,dz$$
$$\leq C\frac{\alpha}{1+\alpha\delta}\left(1 + \delta B\left(\frac{1}{\alpha},\delta\right)\right)\frac{t^{\delta+1/\alpha}}{(t^{1/\alpha} + |y-x|)^{d+\alpha-1}},$$

valid for $\delta > -1/\alpha$, where $C > 0$, and $B(\cdot,\cdot)$ is the Euler beta function. We obtain for $k \geq 1$

$$|V_k(t,x,y)| \leq \frac{(2N)^{k+1}(C\|a\|)^k}{\alpha}\frac{1}{k!}\frac{t^{k/\alpha}}{(t^{1/\alpha} + |y-x|)^{d+\alpha-1}}\prod_{n=1}^{k-1}\left(1 + \frac{n}{\alpha}B\left(\frac{1}{\alpha},\frac{n}{\alpha}\right)\right).$$

Note, that $r_k = \frac{(2NC\|a\|t^{1/\alpha})^k}{k!}\prod_{n=1}^{k-1}\left(1 + \frac{n}{\alpha}B\left(\frac{1}{\alpha},\frac{n}{\alpha}\right)\right)$ is positive and the relation

$$\lim_{k\to\infty}\frac{r_{k+1}}{r_k} = \lim_{k\to\infty}\frac{2NC\|a\|t^{1/\alpha}}{k+1}\left(1 + \frac{k}{\alpha}B\left(\frac{1}{\alpha},\frac{k}{\alpha}\right)\right) = 0$$

is true. Therefore, the series on the right hand side of (10) converges uniformly in $x \in \mathbb{R}^d$, $y \in \mathbb{R}^d$ and locally uniformly in $t > 0$. Thus, the function V, given by the equality (10), is a solution of the equation (9). In addition, the following inequality

$$|V(t,x,y)| \leq C_T\frac{1}{(t^{1/\alpha} + |y-x|)^{d+\alpha-1}} \tag{12}$$

is proved for $x \in \mathbb{R}^d$, $y \in \mathbb{R}^d$ and $0 < t \leq T$, where C_T is a positive constant that may be depended on $T > 0$.

Remark. *The constructed function $V(t,x,y)$ is the unique solution of equation* (9) *in the class of functions that satisfy inequality* (12).

Define the function $G(t,x,y)$ by the equality (8) where the function $V(t,x,y)$ is defined in (10). Then we can perform the following calculations

$$\begin{aligned}(\mathbf{B}_x G(t,x,y), e(x)) &= V_0(t,x,y) + \int_0^t d\tau \int_{\mathbb{R}^d} V_0(t-\tau,x,z)V(\tau,z,y)|a(z)|\,dz \\ &= V(t,x,y).\end{aligned}$$

We here took the possibility of applying of the operator $\mathbf{B}$ under integral, which is proved in the following Lemma.

Lemma. *The equality*

$$\mathbf{B}_x \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,z)V(\tau,z,y)|a(z)|\,dz = \int_0^t d\tau \int_{\mathbb{R}^d} \mathbf{B}_x g(t-\tau,x,z)V(\tau,z,y)|a(z)|\,dz$$

is true.

Proof. Let us consider a set of operators $\{\mathbf{B}^\varepsilon : \varepsilon > 0\}$ that act on a continuously differentiable bounded function $(\varphi(x))_{x\in\mathbb{R}^d}$ according to the following rule

$$(\mathbf{B}^\varepsilon \varphi)(x) = \frac{2c}{\alpha\varkappa} \int_{|u|\geq\varepsilon} \frac{\varphi(x+u)-\varphi(x)}{|u|^{d+\alpha}}\, y\, dy.$$

It is clear that $\lim\limits_{\varepsilon\to 0+} (\mathbf{B}^\varepsilon\varphi)(x) = (\mathbf{B}\varphi)(x)$ for all functions φ, described above, and $x \in \mathbb{R}^d$.

The inequalities (11) and (12) allow us to assert that

$$\begin{aligned}&\left| \frac{u}{|u|^{d+\alpha}} (g(t-\tau,x+u,z) - g(t-\tau,x,z))V(\tau,z,y)|a(z)| \right| \\ &\leq \frac{const}{|u|^{d+\alpha-1}} \left(\frac{t-\tau}{((t-\tau)^{1/\alpha} + |z-x-u|)^{d+\alpha}} + \frac{t-\tau}{((t-\tau)^{1/\alpha} + |z-x|)^{d+\alpha}} \right) \\ &\times \frac{1}{(\tau^{1/\alpha} + |y-z|)^{d+\alpha-1}}.\end{aligned}$$

It is easy to see that the right hand side of this inequality is the integrable function with respect to (u,τ,z) on the set $\{|u|\geq\varepsilon\}\times(0;t)\times\mathbb{R}^d$ for all $t>0$ and $x\in\mathbb{R}^d$, $y\in\mathbb{R}^d$. Here we used the results of [2, Lemma 5], where it is proved that

$$\begin{aligned}&\int_0^t d\tau \int_{\mathbb{R}^d} \frac{(t-\tau)^{\beta/\alpha}}{((t-\tau)^{1/\alpha}+|z-x|)^{d+\alpha+k}} \frac{\tau^{\gamma/\alpha}}{(\tau^{1/\alpha}+|y-z|)^{d+\alpha+l}}\, dz \\ &\leq C\left[B\left(\frac{\beta-k}{\alpha}, 1+\frac{\gamma}{\alpha}\right) t^{\frac{\beta+\gamma-k}{\alpha}} \frac{1}{(t^{1/\alpha}+|y-x|)^{d+\alpha+l}} \right. \\ &\left. + B\left(1+\frac{\beta}{\alpha}, \frac{\gamma-l}{\alpha}\right) t^{\frac{\beta+\gamma-l}{\alpha}} \frac{1}{(t^{1/\alpha}+|y-x|)^{d+\alpha+k}} \right]\end{aligned} \tag{13}$$

for $-\alpha<k<\beta$, $-\alpha<l<\gamma$ and $C>0$, which depends only on d, α, k and l.

Therefore, we obtain the following equality

$$\mathbf{B}^\varepsilon_x \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,z)V(\tau,z,y)|a(z)|\,dz = \int_0^t d\tau \int_{\mathbb{R}^d} \mathbf{B}^\varepsilon_x g(t-\tau,x,z)V(\tau,z,y)|a(z)|\,dz, \tag{14}$$

using the Fubini theorem.

The inequalities (12), (13) and $|\mathbf{B}_x g(t,x,y)| \le \dfrac{const}{(t^{1/\alpha}+|y-x|)^{d+\alpha-1}}$ allow us to assert that the integral $\int_0^t d\tau \int_{\mathbb{R}^d} \mathbf{B}_x g(t-\tau,x,z)V(\tau,z,y)|a(z)|\,dz$ exists. Now we have to pass to the limit with $\varepsilon \to 0+$ in the equality (14) to complete the proof of Lemma. □

We have thus got that the function $G(t,x,y)$ is the perturbation of the transition probability density $g(t,x,y)$ of a symmetric stable process.

Considering estimates (12), (11) and inequality (13), we can write for $t \in (0;T]$, $x \in \mathbb{R}^d$, $y \in \mathbb{R}^d$

$$\begin{aligned} |G(t,x,y)| &\le N\frac{t}{(t^{1/\alpha}+|y-x|)^{d+\alpha}} \\ &+ NC_T\|a\| \int_0^t d\tau \int_{\mathbb{R}^d} \frac{t-\tau}{((t-\tau)^{1/\alpha}+|z-x|)^{d+\alpha}} \frac{1}{(\tau^{1/\alpha}+|y-z|)^{d+\alpha-1}}\,dz \\ &\le \frac{Kt}{(t^{1/\alpha}+|y-x|)^{d+\alpha-1}}\left(1+\frac{1+t^{1/\alpha}}{t^{1/\alpha}+|y-x|}\right), \end{aligned}$$

where K is a positive constant, which depends on T, α, c, $\|a\|$ and d. Note that the right hand side of the last inequality can be estimated from above by the following expression

$$\frac{\hat{K}t^{1-1/\alpha}}{(t^{1/\alpha}+|y-x|)^{d+\alpha-1}} \le \hat{K}t^{-d/\alpha},$$

where $\hat{K} = (2T^{1/\alpha}+1)K$.

2 THE FUNDAMENTAL SOLUTION OF THE CAUCHY PROBLEM

It is known (see [2]) that the function $g(t,x,y)$ is the fundamental solution of the Cauchy problem (3) and, in addition, the function

$$u(t,x) = \int_{\mathbb{R}^d} \varphi(y)g(t,x,y)\,dy + \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,y)f(\tau,y)\,dy$$

is the solution of the Cauchy problem

$$\begin{aligned} &\frac{\partial u(t,x)}{\partial t} = \mathbf{A}_x u(t,x) + f(t,x), \quad t>0,\ x\in\mathbb{R}^d, \\ &u(0+,x) = \varphi(x), \quad x \in \mathbb{R}^d, \end{aligned} \tag{15}$$

for any bounded continuous functions $(\varphi(x))_{x\in\mathbb{R}^d}$ and $(f(t,x))_{t>0,x\in\mathbb{R}^d}$. Moreover, this solution is unique in the class of functions that vanish as $|x|\to\infty$.

Thus, the function

$$\begin{aligned} U(t,x) &= \int_{\mathbb{R}^d} \varphi(y)G(t,x,y)\,dy \\ &= \int_{\mathbb{R}^d} \varphi(y)g(t,x,y)\,dy + \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,y)\int_{\mathbb{R}^d} V(\tau,y,z)\varphi(z)\,dz|a(y)|\,dy \end{aligned}$$

is the unique (in the class of functions that tends to zero at infinity) solution of the Cauchy problem (15) with $f(t,x) = \int_{\mathbb{R}^d} V(t,x,z)\varphi(z)\,dz|a(x)|$.

Now we note that $V(t,x,y) = (\mathbf{B}_x G(t,x,y), e(x))$. Then

$$f(t,x) = \int_{\mathbb{R}^d} (\mathbf{B}_x G(t,x,z), a(x))\varphi(z)\,dz = (a(x), \mathbf{B}_x U(t,x)),$$

and the function $U(t,x)$ is a solution of the Cauchy problem for the equation (4) with bounded continuous function $a(x)$ and operators $\mathbf{A}$ and $\mathbf{B}$ defined by equalities (1) and (5) respectively.

Let us prove that the function $G(t,x,y)$ satisfies the equation of Kolmogorov-Chapman

$$G(t+s,x,y) = \int_{\mathbb{R}^d} G(s,x,z)G(t,z,y)\,dz \tag{16}$$

for all $s>0$, $t>0$, $x\in\mathbb{R}^d$, $y\in\mathbb{R}^d$. Note, the function $g(t,x,y)$ satisfies the equation (16).

Let $(\varphi(x))_{x\in\mathbb{R}^d}$ be a continuous bounded function. Put $U(s,x,\varphi) = \int_{\mathbb{R}^d} G(s,x,y)\varphi(y)\,dy$, $u(s,x,\varphi) = \int_{\mathbb{R}^d} g(s,x,y)\varphi(y)\,dy$ and $W(s,x,\varphi) = \int_{\mathbb{R}^d} V(s,x,y)\varphi(y)\,dy$.

Note, that the function $W(t,x,\varphi)$ is the unique solution of the following equation

$$W(t,x,\varphi) = W_0(t,x,\varphi) + \int_0^t d\tau \int_{\mathbb{R}^d} V_0(t-\tau,x,z)W(\tau,z,\varphi)|a(z)|\,dz, \tag{17}$$

where $W_0(s,x,\varphi) = \int_{\mathbb{R}^d} V_0(s,x,y)\varphi(y)\,dy$.

Then the function $U(s,x,\varphi)$ can be given by the equality (see (5))

$$U(t,x,\varphi) = u(t,x,\varphi) + \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,x,z)W(\tau,z,\varphi)|a(z)|\,dz.$$

Now, let us find the function $U(t+s,x,\varphi)$. We have

$$\begin{aligned} U(t+s,x,\varphi) =& u(t+s,x,\varphi) + \int_0^{t+s} d\tau \int_{\mathbb{R}^d} g(t+s-\tau,x,z)W(\tau,z,\varphi)|a(z)|\,dz \\ &= \int_{\mathbb{R}^d} g(s,x,y)u(t,y,\varphi)\,dy \\ &+ \int_{\mathbb{R}^d} g(s,x,y)\,dy \int_0^t d\tau \int_{\mathbb{R}^d} g(t-\tau,y,z)W(\tau,z,\varphi)|a(z)|\,dz \\ &+ \int_t^{s+t} d\tau \int_{\mathbb{R}^d} g(t+s-\tau,x,z)W(\tau,z,\varphi)|a(z)|\,dz \\ &= \int_{\mathbb{R}^d} g(s,x,y)U(t,y,\varphi)\,dy \\ &+ \int_0^s d\tau \int_{\mathbb{R}^d} g(s-\tau,x,z)W(t+\tau,z,\varphi)|a(z)|\,dz. \end{aligned}$$

Therefore, the function $W_t(s,x,\varphi) = W(t+s,x,\varphi)$ satisfies the equation (17), where the function φ is replaced by $U(t,\cdot,\varphi)$. Then $W(t+s,x,\varphi) = W(s,x,U(t,\cdot,\varphi))$ and we arrive at the equality $U(t+s,x,\varphi) = U(s,x,U(t,\cdot,\varphi))$ or, what is the same,

$$\begin{aligned} \int_{\mathbb{R}^d} G(t+s,x,y)\varphi(y)\,dy &= \int_{\mathbb{R}^d} G(s,x,z) \int_{\mathbb{R}^d} G(t,z,y)\varphi(y)\,dy\,dz \\ &= \int_{\mathbb{R}^d} \varphi(y)\,dy \int_{\mathbb{R}^d} G(s,x,z)G(t,z,y)\,dz. \end{aligned}$$

Then the relation (16) is proved because the function φ is an arbitrary bounded continuous one.

Next, we get $\int_{\mathbb{R}^d} G(t,x,y)\,dy = 1$ from (8) and (9), because there are obvious equalities

$$\int_{\mathbb{R}^d} g(t,x,y)\,dy = 1 \quad \text{and} \quad \int_{\mathbb{R}^d} V_0(t,x,y)\,dy = \left(\mathbf{B}_x \int_{\mathbb{R}^d} g(t,x,y)\,dy, e(x)\right) = 0$$

for all $t > 0$, $x \in \mathbb{R}^d$, and the uniqueness of the solution of equation (9) leads us to the identity $\int_{\mathbb{R}^d} V(t,x,y)\,dy \equiv 0$.

Unfortunately, we can not guarantee non-negativity of the function $G(t,x,y)$ and the existence of a Markov process with the generating operator $\mathbf{A} + (a(\cdot), \mathbf{B})$.

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Осипчук М.М. *Про деяке збурення стійкого процесу та розв'язки задачі Коші для одного класу псевдо-диференціальних рівнянь.* // Карпатські матем. публ. — 2015. — Т.7, №1. — С. 101–107.

З допомогою методу теорії збурень знайдено фундаментальний розв'язок деякого класу псевдо-диференціальних рівнянь. Розглянуто симетричний α-стійкий процес в багатовимірному евклідовому просторі. Його генератор $\mathbf{A}$ є псевдо-диференціальним оператором чий символ задається функцією $-c|\lambda|^\alpha$, де $\alpha \in (1,2)$ і $c > 0$ задані сталі. Векторнозначний оператор $\mathbf{B}$ має символ $2ic|\lambda|^{\alpha-2}\lambda$. Побудовано фундаментальний розв'язок рівняння $u_t = (\mathbf{A} + (a(\cdot), \mathbf{B}))u$ з неперервною обмеженою векторнозначною функцією a.

Ключові слова і фрази: стійкий процес, задача Коші, псевдо-диференціальне рівняння, щільність ймовірності переходу.