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STABILITY OF TRIPLED FIXED POINT ITERATION PROCEDURES FOR MIXED MONOTONE MAPPINGS

Recently, Berinde and Borcut [11] introduced the concept of tripled fixed point and by now, there are several researches on this subject, in partially ordered metric spaces and in cone metric spaces.

In this paper we introduce the notion of stability definition of tripled fixed point iteration procedures and establish stability results for mixed monotone mappings which satisfy various contractive conditions. Our results extend and complete some existing results in the literature. An illustrative example is also given.

Key words and phrases: tripled fixed point, stability, mixed monotone operator, contractive condition.

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INTRODUCTION

Banach-Caccioppoli-Picard Principle was applied on partially ordered complete metric spaces by Ran and Reueings [34] and starting from their results, Bhaskar and Lakshmikantham [12] extend this theory to partially ordered produced metric spaces and introduce the concept of coupled fixed point for mixed-monotone operators of Picard type, obtaining results involving the existence, the existence and the uniqueness of the coincidence points for mixed-monotone operators $T : X^2 \rightarrow X$ in the presence of a contraction type condition.

This concept of coupled fixed points in partially ordered metric and cone metric spaces have been studied by several authors, including Abbas, Ali Khan and Radenovic [1], Berinde [5–7], Choudhury and Kundu [17], Ciric and Lakshmikantham [18], Karapinar [23], Lakshmikantham and Ciric [24], Olatinwo [25], Sabetghadam, Masiha and Sanatpour [37].

Recently, Berinde and Borcut [11, 16] obtained extensions to the concept of tripled fixed points and tripled coincidence fixed points and also obtained tripled fixed points theorems and tripled coincidence fixed points theorems for contractive type mappings in partially ordered metric spaces. Research on tripled fixed point was continued by Abbas, Aydi and Karapinar [2], Aydi and Karapinar [4], Amini-Harandi [3], Borcut [13–15], Rao and Kishore [34].

In the case of fixed points of an operator $T : X^2 \rightarrow X$, the stability of a fixed point iterative procedures was first studied by Ostrowski [33] in the case of Banach contraction mappings and this subject was later developed for certain contractive definitions by several authors (see Harder and Hicks [19], Rhoades [35,36], Osilike [30,31], Osilike and Udomene [32], Berinde [8–

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10], Jachymski [22], Olatinwo [26, 27], Imoru and Olatinwo [20], Imoru, Olatinwo and Owojori [21, 29] etc.).

On the other hand, adapting the concept of stability from fixed point iterative procedures, Olatinwo [28] studied the stability of the coupled fixed point iterative procedures using some contractive conditions for which the existence of a unique coupled fixed point has been established by Sabetghadam, Masiha and Sanatpour [37].

Our aim in this paper is to introduce the concept of stability for tripled fixed point iterative procedures and to establish stability results for mixed monotone mappings satisfying various contractive conditions by extension from coupled fixed points to tripled fixed points of contractive conditions employed by Olatinwo [28].

1 PRELIMINARIES

Let (X, \leq) be a partially ordered set and d be a metric on X such that (X, d) is a complete metric space. Berinde and Borcut [11] endowed the product space X^3 with the following partial order

$$(u, v, w) \leq (x, y, z) \iff x \geq u, y \leq v, z \geq w, \quad (u, v, w), (x, y, z) \in X^3.$$

Definition 1 ([11]). Let (X, \leq) be a partially ordered set and $T : X^3 \rightarrow X$ be a mapping. We say that T has the mixed monotone property if $T(x, y, z)$ is monotone nondecreasing in x , monotone nonincreasing in y and monotone nondecreasing in z , that is, for any $x, y, z \in X$,

$$\begin{aligned} x_1 \leq x_2 &\implies T(x_1, y, z) \leq T(x_2, y, z), & x_1, x_2 \in X, \\ y_1 \leq y_2 &\implies T(x, y_1, z) \geq T(x, y_2, z), & y_1, y_2 \in X, \\ z_1 \leq z_2 &\implies T(x, y, z_1) \leq T(x, y, z_2), & z_1, z_2 \in X. \end{aligned}$$

Definition 2 ([11]). An element $(x, y, z) \in X^3$ is called tripled fixed point of $T : X^3 \rightarrow X$, if

$$T(x, y, z) = x, \quad T(y, x, y) = y, \quad T(z, y, x) = z.$$

A mapping $T : X^3 \rightarrow X$ is said to be a (k, μ, ρ) -contraction if and only if there exists three constants $k \geq 0, \mu \geq 0, \rho \geq 0, k + \mu + \rho < 1$, such that $\forall x, y, z, u, v, w \in X$,

$$d(T(x, y, z), T(u, v, w)) \leq kd(x, u) + \mu d(y, v) + \rho d(z, w). \quad (1)$$

In relation to (1), we introduce some new contractive conditions.

Let (X, d) be a metric space. For a map $T : X^3 \rightarrow X$ there exist $a_1, a_2, a_3, b_1, b_2, b_3 \geq 0$, with $a_1 + a_2 + a_3 < 1, b_1 + b_2 + b_3 < 1$, such that $\forall x, y, z, u, v, w \in X$ we introduce the following definitions of contractive conditions:

$$(i) \quad d(T(x, y, z), T(u, v, w)) \leq a_1 d(T(x, y, z), x) + b_1 d(T(u, v, w), u); \quad (2)$$

$$d(T(y, x, y), T(v, u, v)) \leq a_2 d(T(y, x, y), y) + b_2 d(T(v, u, v), v); \quad (3)$$

$$d(T(w, y, x), T(z, v, u)) \leq a_3 d(T(z, y, x), z) + b_3 d(T(w, v, u), w); \quad (4)$$

$$(ii) \quad d(T(x, y, z), T(u, v, w)) \leq a_1 d(T(x, y, z), u) + b_1 d(T(u, v, w), x); \quad (5)$$

$$d(T(y, x, y), T(v, u, v)) \leq a_2 d(T(y, x, y), v) + b_2 d(T(v, u, v), y); \quad (6)$$

$$d(T(w, y, x), T(z, v, u)) \leq a_3 d(T(z, y, x), w) + b_3 d(T(w, v, u), z). \quad (7)$$

Let $A, B \in M_{(m,n)}(\mathbb{R})$ be two matrices. We write $A \leq B$, if $a_{ij} \leq b_{ij}$ for all $i = \overline{1, m}, j = \overline{1, n}$.

In order to prove our main stability result in this paper we give the next

Lemma 1 ([8]). *Let $\{a_n\}, \{b_n\}$ be sequences of nonnegative numbers and h be a constant, such that $0 \leq h < 1$ and*

$$a_{n+1} \leq ha_n + b_n, \quad n \geq 0.$$

If $\lim_{n \rightarrow \infty} b_n = 0$, then $\lim_{n \rightarrow \infty} a_n = 0$.

We also give the next result which extends Lemma 1 to vector sequences, where inequalities between vectors means inequality on its elements.

Lemma 2. *Let $\{u_n\}, \{v_n\}, \{w_n\}$ be sequences of nonnegative real numbers. Consider a matrix $A \in M_{3,3}(\mathbb{R})$ with nonnegative elements, such that*

$$\begin{pmatrix} u_{n+1} \\ v_{n+1} \\ w_{n+1} \end{pmatrix} \leq A \cdot \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} + \begin{pmatrix} \varepsilon_n \\ \delta_n \\ \gamma_n \end{pmatrix}, \quad n \geq 0, \tag{8}$$

with

(i) $\lim_{n \rightarrow \infty} A^n = O_3$;

(ii) $\sum_{k=0}^{\infty} \varepsilon_k < \infty, \sum_{k=0}^{\infty} \delta_k < \infty$ and $\sum_{k=0}^{\infty} \gamma_k < \infty$.

If $\lim_{n \rightarrow \infty} \begin{pmatrix} \varepsilon_n \\ \delta_n \\ \gamma_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, then $\lim_{n \rightarrow \infty} \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Proof. For $A = 0 \in M_{(3,3)}$, the conclusion is obvious.

We rewrite (8) with $n = k$ and sum the inequalities obtained for $k = 0, 1, 2, \dots, n$. After doing all cancellations, we obtain

$$\begin{pmatrix} u_{n+1} \\ v_{n+1} \\ w_{n+1} \end{pmatrix} \leq A^{n+1} \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix} + \sum_{k=0}^n A^k \begin{pmatrix} \varepsilon_{n-k} \\ \delta_{n-k} \\ \gamma_{n-k} \end{pmatrix}. \tag{9}$$

From (ii) it follows that the sequences of partial sums $\{E_n\}, \{\Delta_n\}$ and $\{\Gamma_n\}$, given respectively by $E_n = \varepsilon_0 + \varepsilon_1 + \dots + \varepsilon_n, \Delta_n = \delta_0 + \delta_1 + \dots + \delta_n$ and $\Gamma_n = \gamma_0 + \gamma_1 + \dots + \gamma_n$, for $n \geq 0$, converge respectively to some $E \geq 0, \Delta \geq 0$ and $\Gamma \geq 0$ and hence, they are bounded.

Let $M > 0$ be such that $\begin{pmatrix} E_n \\ \Delta_n \\ \Gamma_n \end{pmatrix} \leq M \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \forall n \geq 0$. By (i) we have that $\forall e > 0$, there

exists $N = N(e)$ such that $A^n \leq \frac{e}{2M} \cdot I_3, \forall n \geq N, M > 0$.

We can write

$$\begin{aligned} \sum_{k=0}^n A^k \begin{pmatrix} \varepsilon_{n-k} \\ \delta_{n-k} \\ \gamma_{n-k} \end{pmatrix} &= A^n \begin{pmatrix} \varepsilon_0 \\ \delta_0 \\ \gamma_0 \end{pmatrix} + \dots + A^N \begin{pmatrix} \varepsilon_{n-N} \\ \delta_{n-N} \\ \gamma_{n-N} \end{pmatrix} \\ &\quad + A^{N-1} \begin{pmatrix} \varepsilon_{n-N+1} \\ \delta_{n-N+1} \\ \gamma_{n-N+1} \end{pmatrix} + \dots + I_3 \begin{pmatrix} \varepsilon_n \\ \delta_n \\ \gamma_n \end{pmatrix}. \end{aligned}$$

But

$$\begin{aligned} A^n \begin{pmatrix} \varepsilon_0 \\ \delta_0 \\ \gamma_0 \end{pmatrix} + \cdots + A^N \begin{pmatrix} \varepsilon_{n-N} \\ \delta_{n-N} \\ \gamma_{n-N} \end{pmatrix} &\leq \frac{e}{2M} \cdot I_3 \left[\begin{pmatrix} \varepsilon_0 \\ \delta_0 \\ \gamma_0 \end{pmatrix} + \cdots + \begin{pmatrix} \varepsilon_{n-N} \\ \delta_{n-N} \\ \gamma_{n-N} \end{pmatrix} \right] \\ &= \frac{e}{2M} \cdot I_3 \begin{pmatrix} E_{n-N} \\ \Delta_{n-N} \\ \Gamma_{n-N} \end{pmatrix} \leq \frac{e}{2M} \cdot I_3 \cdot M \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{e}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \end{aligned}$$

for all $n \geq N$. On the other hand, if we denote $A' = \max \{I_3, A, \dots, A^{N-1}\}$, we obtain

$$\begin{aligned} A^{N-1} \begin{pmatrix} \varepsilon_{n-N+1} \\ \delta_{n-N+1} \\ \gamma_{n-N+1} \end{pmatrix} + \cdots + I_3 \begin{pmatrix} \varepsilon_n \\ \delta_n \\ \gamma_n \end{pmatrix} &\leq A' \left[\begin{pmatrix} \varepsilon_{n-N+1} \\ \delta_{n-N+1} \\ \gamma_{n-N+1} \end{pmatrix} + \cdots + \begin{pmatrix} \varepsilon_n \\ \delta_n \\ \gamma_n \end{pmatrix} \right] \\ &= A' \begin{pmatrix} E_n - E_{n-N} \\ \Delta_n - \Delta_{n-N} \\ \Gamma_n - \Gamma_{n-N} \end{pmatrix}. \end{aligned}$$

As N is fixed, then $\lim_{n \rightarrow \infty} E_n = \lim_{n \rightarrow \infty} E_{n-N} = E$, $\lim_{n \rightarrow \infty} \Delta_n = \lim_{n \rightarrow \infty} \Delta_{n-N} = \Delta$, and $\lim_{n \rightarrow \infty} \Gamma_n = \lim_{n \rightarrow \infty} \Gamma_{n-N} = \Gamma$, which shows that there exists a positive integer k such that

$$A' \begin{pmatrix} E_n - E_{n-N} \\ \Delta_n - \Delta_{n-N} \\ \Gamma_n - \Gamma_{n-N} \end{pmatrix} < \frac{e}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \forall n \geq k.$$

Now, for $m = \max \{k, N\}$, we get

$$A^n \begin{pmatrix} \varepsilon_0 \\ \delta_0 \\ \gamma_0 \end{pmatrix} + \cdots + I_3 \begin{pmatrix} \varepsilon_n \\ \delta_n \\ \gamma_n \end{pmatrix} < e \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \forall n \geq m,$$

and therefore, $\lim_{n \rightarrow \infty} \sum_{k=0}^n A^k \begin{pmatrix} \varepsilon_{n-k} \\ \delta_{n-k} \\ \gamma_{n-k} \end{pmatrix} = 0$.

Now, by letting the limit in (9), as $\lim_{n \rightarrow \infty} A^n = 0$, we get

$$\lim_{n \rightarrow \infty} \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

as required. □

2 STABILITY RESULTS

Let (X, d) be a metric space and $T : X^3 \rightarrow X$ a mapping. For $(x_0, y_0, z_0) \in X^3$ the sequence $\{(x_n, y_n, z_n)\} \subset X^3$ defined by

$$x_{n+1} = T(x_n, y_n, z_n), \quad y_{n+1} = T(y_n, x_n, y_n), \quad z_{n+1} = T(z_n, y_n, x_n), \quad (10)$$

with $n = 0, 1, 2, \dots$, is said to be a *tripled fixed point iterative procedure*.

We give the following definition of stability with respect to T , in metric spaces, relative to tripled fixed points iterative procedures.

Definition 3. Let (X, d) be a complete metric space and

$$\text{Fix}_t(T) = \left\{ (x^*, y^*, z^*) \in X^3 \mid T(x^*, y^*, z^*) = x^*, T(y^*, x^*, y^*) = y^*, T(z^*, y^*, x^*) = z^* \right\}$$

is the set of tripled fixed points of T .

Let $\{(x_n, y_n, z_n)\} \subset X^3$ be the sequence generated by the iterative procedure defined by (10), where $(x_0, y_0, z_0) \in X^3$ is the initial value, which converges to a tripled fixed point (x^*, y^*, z^*) of T .

Let $\{(u_n, v_n, w_n)\} \subset X^3$ be an arbitrary sequence. For all $n = 0, 1, 2, \dots$ we set

$$\varepsilon_n = d(u_{n+1}, T(u_n, v_n, w_n)), \quad \delta_n = d(v_{n+1}, T(v_n, u_n, v_n)), \quad \gamma_n = d(w_{n+1}, T(w_n, v_n, u_n)).$$

Then the tripled fixed point iterative procedure defined by (10) is T -stable or stable with respect to T , if and only if

$$\lim_{n \rightarrow \infty} (\varepsilon_n, \delta_n, \gamma_n) = 0_{\mathbb{R}^3} \quad \text{implies that} \quad \lim_{n \rightarrow \infty} (u_n, v_n, w_n) = (x^*, y^*, z^*).$$

Theorem 1. Let (X, \leq) be a partially ordered set. Suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T : X^3 \rightarrow X$ be a continuous mapping having the mixed monotone property on X and satisfying (1).

If there exists $x_0, y_0, z_0 \in X$ such that

$$x_0 \leq T(x_0, y_0, z_0), \quad y_0 \geq T(y_0, x_0, y_0) \quad \text{and} \quad z_0 \leq T(z_0, y_0, x_0),$$

then there exist $x^*, y^*, z^* \in X$ such that

$$x^* = T(x^*, y^*, z^*), \quad y^* = T(y^*, x^*, y^*) \quad \text{and} \quad z^* = T(z^*, y^*, x^*).$$

Assume that for every $(x, y, z), (x_1, y_1, z_1) \in X^3$, there exists $(u, v, w) \in X^3$ that is comparable to (x, y, z) and (x_1, y_1, z_1) . For $(x_0, y_0, z_0) \in X^3$, let $\{(x_n, y_n, z_n)\} \subset X^3$ be the tripled fixed point iterative procedure defined by (10). Then the tripled fixed point iterative procedure is stable with respect to T .

Proof. From the suppositions of the hypothesis, Berinde and Borcut [11] proved the existence and uniqueness of the tripled fixed point and now, using these results, we can study the stability of the tripled fixed point iterative procedures.

Let $\{(x_n, y_n, z_n)\} \subset X^3$, $\varepsilon_n = d(u_{n+1}, T(u_n, v_n, w_n))$, $\delta_n = d(v_{n+1}, T(v_n, u_n, v_n))$ and $\gamma_n = d(w_{n+1}, T(w_n, v_n, u_n))$.

Assume also that $\lim_{n \rightarrow \infty} \varepsilon_n = \lim_{n \rightarrow \infty} \delta_n = \lim_{n \rightarrow \infty} \gamma_n = 0$ in order to establish that $\lim_{n \rightarrow \infty} u_n = x^*$, $\lim_{n \rightarrow \infty} v_n = y^*$ and $\lim_{n \rightarrow \infty} w_n = z^*$.

Therefore, using the (k, μ, ρ) -contraction condition (1), we obtain

$$\begin{aligned} d(u_{n+1}, x^*) &\leq d(u_{n+1}, T(u_n, v_n, w_n)) + d(T(u_n, v_n, w_n), x^*) \\ &= d(T(u_n, v_n, w_n), T(x^*, y^*, z^*)) + \varepsilon_n \\ &\leq kd(u_n, x^*) + \mu d(v_n, y^*) + \rho d(w_n, z^*) + \varepsilon_n, \end{aligned} \tag{11}$$

$$\begin{aligned} d(v_{n+1}, y^*) &\leq d(v_{n+1}, T(v_n, u_n, v_n)) + d(T(v_n, u_n, v_n), y^*) \\ &= d(T(v_n, u_n, v_n), T(y^*, x^*, y^*)) + \delta_n \\ &\leq kd(v_n, y^*) + \mu d(u_n, x^*) + \rho d(v_n, y^*) + \delta_n, \end{aligned} \tag{12}$$

$$\begin{aligned}
d(w_{n+1}, z^*) &\leq d(w_{n+1}, T(w_n, v_n, u_n)) + d(T(w_n, v_n, u_n), z^*) \\
&= d(T(w_n, v_n, u_n), T(z^*, y^*, x^*)) + \gamma_n \\
&\leq kd(w_n, z^*) + \mu d(v_n, y^*) + \rho d(u_n, x^*) + \gamma_n.
\end{aligned} \tag{13}$$

From (11), (12) and (13), we obtain

$$\begin{pmatrix} d(u_{n+1}, x^*) \\ d(v_{n+1}, y^*) \\ d(w_{n+1}, z^*) \end{pmatrix} \leq \begin{pmatrix} k & \mu & \rho \\ \mu & k + \rho & 0 \\ \rho & \mu & k \end{pmatrix} \cdot \begin{pmatrix} d(u_n, x^*) \\ d(v_n, y^*) \\ d(w_n, z^*) \end{pmatrix} + \begin{pmatrix} \varepsilon_n \\ \delta_n \\ \gamma_n \end{pmatrix}.$$

We denote $A := \begin{pmatrix} k & \mu & \rho \\ \mu & k + \rho & 0 \\ \rho & \mu & k \end{pmatrix}$, where $0 \leq k + \mu + \rho < 1$, as in (1).

In order to apply Lemma 2, we need that $A^n \rightarrow 0$, as $n \rightarrow \infty$. Simplifying the writing,

$$A := \begin{pmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \\ g_1 & h_1 & l_1 \end{pmatrix}, \text{ where } a_1 + b_1 + c_1 = d_1 + e_1 + f_1 = g_1 + h_1 + l_1 = k + \mu + \rho < 1.$$

Then

$$\begin{aligned}
A^2 &= \begin{pmatrix} k & \mu & \rho \\ \mu & k + \rho & 0 \\ \rho & \mu & k \end{pmatrix} \cdot \begin{pmatrix} k & \mu & \rho \\ \mu & k + \rho & 0 \\ \rho & \mu & k \end{pmatrix} \\
&= \begin{pmatrix} k^2 + \mu^2 + \rho^2 & 2k\mu + 2\mu\rho & 2k\rho \\ 2k\mu + \rho\mu & k^2 + \mu^2 + \rho^2 + 2k\rho & \mu\rho \\ 2k\rho + \mu^2 & 2k\mu + 2\rho\mu & k^2 + \rho^2 \end{pmatrix} := \begin{pmatrix} a_2 & b_2 & c_2 \\ d_2 & e_2 & f_2 \\ g_2 & h_2 & l_2 \end{pmatrix},
\end{aligned}$$

where $a_2 + b_2 + c_2 = d_2 + e_2 + f_2 = g_2 + h_2 + l_2 = (k + \mu + \rho)^2 < k + \mu + \rho < 1$.

Now, we prove by induction that

$$A^n = \begin{pmatrix} a_n & b_n & c_n \\ d_n & e_n & f_n \\ g_n & h_n & l_n \end{pmatrix},$$

where

$$a_n + b_n + c_n = d_n + e_n + f_n = g_n + h_n + l_n = (k + \mu + \rho)^n < k + \mu + \rho < 1. \tag{14}$$

If we assume that (14) is true for n , then since

$$\begin{aligned}
A^{n+1} &= \begin{pmatrix} a_n & b_n & c_n \\ d_n & e_n & f_n \\ g_n & h_n & l_n \end{pmatrix} \cdot \begin{pmatrix} k & \mu & \rho \\ \mu & k + \rho & 0 \\ \rho & \mu & k \end{pmatrix} \\
&= \begin{pmatrix} ka_n + \mu b_n + \rho c_n & \mu a_n + kb_n + \rho b_n + \mu c_n & \rho a_n + kc_n \\ kd_n + \mu e_n + \rho f_n & \mu d_n + ke_n + \rho e_n + \mu f_n & \rho d_n + kf_n \\ kg_n + \mu h_n + \rho l_n & \mu g_n + kb_n + \rho b_n + \mu h_n & \rho g_n + kh_n \end{pmatrix},
\end{aligned}$$

we have

$$\begin{aligned}
a_{n+1} + b_{n+1} + c_{n+1} &= ka_n + \mu b_n + \rho c_n + \mu a_n + kb_n + \rho b_n + \mu c_n + \rho a_n + kc_n \\
&= (k + \mu + \rho)a_n + (k + \mu + \rho)b_n + (k + \mu + \rho)c_n \\
&= (k + \mu + \rho)(a_n + b_n + c_n) = (k + \mu + \rho)(k + \mu + \rho)^n \\
&= (k + \mu + \rho)^{n+1} < k + \mu + \rho < 1.
\end{aligned}$$

Similarly, we obtain

$$d_{n+1} + e_{n+1} + f_{n+1} = g_{n+1} + b_{n+1} + h_{n+1} = (k + \mu + \rho)^{n+1} < k + \mu + \rho < 1.$$

Therefore, $\lim_{n \rightarrow \infty} A^n = O_3$ and now, having satisfied the conditions of the hypothesis of Lemma 2, we can apply it and we get

$$\lim_{n \rightarrow \infty} \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} = \begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix},$$

so the tripled fixed point iteration procedure defined by (10) is T -stable. \square

Remark 1. Theorem 1 completes the existence theorem of tripled fixed points of Berinde and Borcut [11] with the stability result for the tripled fixed point iterative procedures, using mixed-monotone operators.

Corollary 1. Let (X, \leq) be a partially ordered set. Suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T : X^3 \rightarrow X$ be a continuous mapping having the mixed monotone property on X .

There exists $\kappa \in [0, 1)$, such that T satisfies the following contraction condition

$$d(T(x, y, z), T(u, v, w)) \leq \frac{\kappa}{3} [d(x, u) + d(y, v) + d(z, w)], \quad (15)$$

for each $x, y, z, u, v, w \in X$, with $x \geq u, y \leq v$ and $z \geq w$.

If there exists $x_0, y_0, z_0 \in X$ such that

$$x_0 \leq T(x_0, y_0, z_0), \quad y_0 \geq T(y_0, x_0, y_0) \quad \text{and} \quad z_0 \leq T(z_0, y_0, x_0),$$

then there exist $x^*, y^*, z^* \in X$ such that

$$x^* = T(x^*, y^*, z^*), \quad y^* = T(y^*, x^*, y^*) \quad \text{and} \quad z^* = T(z^*, y^*, x^*).$$

Assume that for every $(x, y, z), (x_1, y_1, z_1) \in X^3$, there exists $(u, v, w) \in X^3$ that is comparable to (x, y, z) and (x_1, y_1, z_1) . For $(x_0, y_0, z_0) \in X^3$, let $\{(x_n, y_n, z_n)\} \subset X^3$ be the tripled fixed point iterative procedure defined by (10). Then, the tripled fixed point iterative procedure is stable with respect to T .

Proof. We apply Theorem 1, for $k = \mu = \rho := \frac{\kappa}{3}$. \square

Remark 2. Corollary 1 completes the existence theorem of tripled fixed points of Berinde and Borcut [11] with the stability result for the tripled fixed point iterative procedures, using mixed-monotone operators.

Theorem 2. Let (X, \leq) be a partially ordered set. Suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T : X^3 \rightarrow X$ be a continuous mapping having the mixed monotone property on X and satisfying (2), (3) and (4).

If there exist $x_0, y_0, z_0 \in X$ such that

$$x_0 \leq T(x_0, y_0, z_0), \quad y_0 \geq T(y_0, x_0, y_0) \quad \text{and} \quad z_0 \leq T(z_0, y_0, x_0),$$

then there exist $x^*, y^*, z^* \in X$ such that

$$x^* = T(x^*, y^*, z^*), \quad y^* = T(y^*, x^*, y^*) \quad \text{and} \quad z^* = T(z^*, y^*, x^*).$$

Assume that for every $(x, y, z), (x_1, y_1, z_1) \in X^3$, there exists $(u, v, w) \in X^3$ that is comparable to (x, y, z) and (x_1, y_1, z_1) . For $(x_0, y_0, z_0) \in X^3$, let $\{(x_n, y_n, z_n)\} \subset X^3$ be the tripled fixed point iterative procedure defined by (10). Then, the tripled fixed point iterative procedure is stable with respect to T .

Proof. Let $\{(x_n, y_n, z_n)\} \subset X^3$, $\varepsilon_n = d(u_{n+1}, T(u_n, v_n, w_n))$, $\delta_n = d(v_{n+1}, T(v_n, u_n, v_n))$ and $\gamma_n = d(w_{n+1}, T(w_n, v_n, u_n))$. Assume also that $\lim_{n \rightarrow \infty} \varepsilon_n = \lim_{n \rightarrow \infty} \delta_n = \lim_{n \rightarrow \infty} \gamma_n = 0$ in order to establish that $\lim_{n \rightarrow \infty} u_n = x^*$, $\lim_{n \rightarrow \infty} v_n = y^*$ and $\lim_{n \rightarrow \infty} w_n = z^*$.

Therefore, using the contraction condition (2), we obtain

$$\begin{aligned} d(u_{n+1}, x^*) &\leq d(u_{n+1}, T(u_n, v_n, w_n)) + d(T(u_n, v_n, w_n), x^*) \\ &= d(T(u_n, v_n, w_n), T(x^*, y^*, z^*)) + \varepsilon_n \\ &\leq a_1 d(T(x^*, y^*, z^*), x^*) + b_1 d(T(u_n, v_n, w_n), u_n) + \varepsilon_n \\ &\leq a_1 d(x^*, x^*) + b_1 d(T(u_n, v_n, w_n), u_{n+1}) + b_1 d(u_{n+1}, x^*) + b_1 d(x^*, u_n) + \varepsilon_n \\ &= a_1 d(x^*, x^*) + b_1 d(u_{n+1}, x^*) + b_1 d(x^*, u_n) + (b_1 + 1)\varepsilon_n. \end{aligned}$$

Hence, $(1 - b_1)d(u_{n+1}, x^*) \leq b_1 d(x^*, u_n) + \varepsilon'_n$, where $\varepsilon'_n := (b_1 + 1)\varepsilon_n + a_1 d(x^*, x^*)$. Passing it to the limit and applying Lemma 1 for $\frac{b_1}{1-b_1} \in [0, 1)$, we obtain that $\lim_{n \rightarrow \infty} u_n = x^*$.

Now, using the contraction condition (3), we obtain

$$\begin{aligned} d(v_{n+1}, y^*) &\leq d(v_{n+1}, T(v_n, u_n, v_n)) + d(T(v_n, u_n, v_n), y^*) \\ &= d(T(v_n, u_n, v_n), T(y^*, x^*, y^*)) + \delta_n \\ &\leq a_2 d(T(y^*, x^*, y^*), y^*) + b_2 d(T(v_n, u_n, v_n), v_n) + \delta_n \\ &\leq a_2 d(y^*, y^*) + b_2 d(T(v_n, u_n, v_n), v_{n+1}) + b_2 d(v_{n+1}, y^*) + b_2 d(y^*, v_n) + \delta_n \\ &= a_2 d(y^*, y^*) + b_2 d(v_{n+1}, y^*) + b_2 d(y^*, v_n) + (b_2 + 1)\delta_n. \end{aligned}$$

So, $(1 - b_2)d(v_{n+1}, y^*) \leq b_2 d(y^*, v_n) + \delta'_n$, where $\delta'_n := (b_2 + 1)\delta_n + a_2 d(y^*, y^*)$. Passing it to the limit and applying Lemma 1 for $\frac{b_2}{1-b_2} \in [0, 1)$, we obtain that $\lim_{n \rightarrow \infty} v_n = y^*$.

Similarly, using the contraction condition (4), we obtain

$$\begin{aligned} d(w_{n+1}, z^*) &\leq d(w_{n+1}, T(z_n, v_n, u_n)) + d(T(z_n, v_n, u_n), z^*) \\ &= d(T(w_n, v_n, u_n), T(z^*, y^*, x^*)) + \gamma_n \\ &\leq a_3 d(T(z^*, y^*, x^*), z^*) + b_3 d(T(w_n, v_n, u_n), w_n) + \gamma_n \\ &\leq a_3 d(z^*, z^*) + b_3 d(T(w_n, v_n, u_n), w_{n+1}) + b_3 d(w_{n+1}, z^*) + b_3 d(z^*, w_n) + \gamma_n \\ &= a_3 d(z^*, z^*) + b_3 d(w_{n+1}, z^*) + b_3 d(z^*, w_n) + (b_3 + 1)\gamma_n. \end{aligned}$$

Therefore, $(1 - b_3)d(w_{n+1}, z^*) \leq b_3 d(z^*, w_n) + \gamma'_n$, where $\gamma'_n := (b_3 + 1)\gamma_n + a_3 d(z^*, z^*)$. Passing it to the limit and applying Lemma 1 for $\frac{b_3}{1-b_3} \in [0, 1)$, we obtain that $\lim_{n \rightarrow \infty} w_n = z^*$ and then we get the conclusion. \square

Theorem 3. Let (X, \leq) be a partially ordered set. Suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T : X^3 \rightarrow X$ be a continuous mapping having the mixed monotone property on X and satisfying (5), (6) and (7).

If there exist $x_0, y_0, z_0 \in X$ such that

$$x_0 \leq T(x_0, y_0, z_0), \quad y_0 \geq T(y_0, x_0, y_0) \quad \text{and} \quad z_0 \leq T(z_0, y_0, x_0),$$

then there exist $x^*, y^*, z^* \in X$ such that

$$x^* = T(x^*, y^*, z^*), \quad y^* = T(y^*, x^*, y^*) \quad \text{and} \quad z^* = T(z^*, y^*, x^*).$$

Assume that for every $(x, y, z), (x_1, y_1, z_1) \in X^3$, there exists $(u, v, w) \in X^3$ that is comparable to (x, y, z) and (x_1, y_1, z_1) . For $(x_0, y_0, z_0) \in X^3$, let $\{(x_n, y_n, z_n)\} \subset X^3$ be the tripled fixed point iterative procedure defined by (10). Then, the tripled fixed point iterative procedure is stable with respect to T .

Proof. Let $\{(x_n, y_n, z_n)\}_{n=0}^\infty \subset X^3$, $\varepsilon_n = d(u_{n+1}, T(u_n, v_n, w_n))$, $\delta_n = d(v_{n+1}, T(v_n, u_n, v_n))$ and $\gamma_n = d(w_{n+1}, T(w_n, v_n, u_n))$.

Assume also that $\lim_{n \rightarrow \infty} \varepsilon_n = \lim_{n \rightarrow \infty} \delta_n = \lim_{n \rightarrow \infty} \gamma_n = 0$ in order to establish that $\lim_{n \rightarrow \infty} u_n = x^*$, $\lim_{n \rightarrow \infty} v_n = y^*$ and $\lim_{n \rightarrow \infty} w_n = z^*$.

Therefore, using the contraction condition (5), we obtain

$$\begin{aligned} d(u_{n+1}, x^*) &\leq d(u_{n+1}, T(u_n, v_n, w_n)) + d(T(u_n, v_n, w_n), x^*) \\ &= d(T(u_n, v_n, w_n), T(x^*, y^*, z^*)) + \varepsilon_n \\ &\leq a_1 d(T(x^*, y^*, z^*), u_n) + b_1 d(T(u_n, v_n, w_n), x^*) + \varepsilon_n \\ &\leq a_1 d(u_n, x^*) + b_1 d(T(u_n, v_n, w_n), u_n) + b_1 d(u_n, x^*) + \varepsilon_n \\ &= (a_1 + b_1) d(u_n, x^*) + \varepsilon_n + b_1 \varepsilon_{n-1}. \end{aligned}$$

Hence, passing it to the limit and applying Lemma 1 for $h := a_1 + b_1 \in [0, 1)$ and for $\varepsilon'_n := \varepsilon_n + b_1 \varepsilon_{n-1} \rightarrow 0$, we obtain that $\lim_{n \rightarrow \infty} u_n = x^*$.

Now, using the contraction condition (6), we obtain

$$\begin{aligned} d(v_{n+1}, y^*) &\leq d(v_{n+1}, T(v_n, u_n, v_n)) + d(T(v_n, u_n, v_n), y^*) \\ &= d(T(v_n, u_n, v_n), T(y^*, x^*, y^*)) + \delta_n \\ &\leq a_2 d(T(y^*, x^*, y^*), v_n) + b_2 d(T(v_n, u_n, v_n), y^*) + \delta_n \\ &\leq a_2 d(v_n, y^*) + b_2 d(T(v_n, u_n, v_n), v_n) + b_2 d(v_n, y^*) + \delta_n \\ &= (a_2 + b_2) d(v_n, y^*) + \delta_n + b_2 \delta_{n-1}. \end{aligned}$$

So, passing it to the limit and applying Lemma 1 for $h := a_2 + b_2 \in [0, 1)$ and for $\delta'_n := \delta_n + b_2 \delta_{n-1} \rightarrow 0$, we get $\lim_{n \rightarrow \infty} v_n = y^*$.

Similarly, using the contraction condition (7), we obtain

$$\begin{aligned} d(w_{n+1}, z^*) &\leq d(w_{n+1}, T(z_n, v_n, u_n)) + d(T(z_n, v_n, u_n), z^*) \\ &= d(T(w_n, v_n, u_n), T(z^*, y^*, x^*)) + \gamma_n \\ &\leq a_3 d(T(z^*, y^*, x^*), w_n) + b_3 d(T(w_n, v_n, u_n), z^*) + \gamma_n \\ &\leq a_3 d(w_n, z^*) + b_3 d(T(w_n, v_n, u_n), w_n) + b_3 d(w_n, z^*) + \gamma_n \\ &= a_3 d(w_n, z^*) + b_3 d(w_n, z^*) + b_3 d(T(w_n, v_n, u_n), w_n) + \gamma_n \\ &= (a_3 + b_3) d(w_n, z^*) + \gamma_n + b_3 \gamma_{n-1}. \end{aligned}$$

Hence, passing it to the limit and applying Lemma 1 for $h := a_3 + b_3 \in [0, 1)$ and for $\gamma'_n := \gamma_n + b_3 \gamma_{n-1} \rightarrow 0$, we obtain that $\lim_{n \rightarrow \infty} w_n = z^*$ and then we get the conclusion. \square

3 ILLUSTRATIVE EXAMPLE

Let (X, d) be a complete metric space, where $X = \mathbb{R}$, $d(x, y) = |x - y|$. Consider a continuous and mixed monotone mapping $T : \mathbb{R}^3 \rightarrow \mathbb{R}$, with $T(x, y, z) = \frac{2x - 2y + 2z + 1}{12}$.

Berinde and Borcut [11] proved the existence and the uniqueness of the tripled fixed point of T , respectively $(x^*, y^*, z^*) = \left(\frac{1}{10}, \frac{1}{10}, \frac{1}{10}\right)$, using $(x_0, y_0, z_0) = \left(\frac{1}{20}, \frac{1}{5}, \frac{1}{20}\right)$.

For $\kappa = \frac{1}{2}$, T satisfies the contraction condition (15), i.e.,

$$d(T(x, y, z), T(u, v, w)) \leq \frac{\kappa}{3} [d(x, u) + d(y, v) + d(z, w)],$$

for each $x, y, z, u, v, w \in X$, with $x \geq u$, $y \leq v$ and $z \geq w$.

We apply Corollary 1 in order to prove the stability of the tripled fixed point iteration procedure.

Let $\{(x_n, y_n, z_n)\} \subset \mathbb{R}^3$ be the sequence generated by the iterative procedure defined by (10), where $(x_0, y_0, z_0) = \left(\frac{1}{20}, \frac{1}{5}, \frac{1}{20}\right) \in \mathbb{R}^3$ is the initial value, which converges to a tripled fixed point $(x^*, y^*, z^*) = \left(\frac{1}{10}, \frac{1}{10}, \frac{1}{10}\right)$ of T .

Let $\{(u_n, v_n, w_n)\} \subset \mathbb{R}^3$ be an arbitrary sequence. For all $n = 0, 1, 2, \dots$ set

$$\varepsilon_n = d(u_{n+1}, T(u_n, v_n, w_n)), \quad \delta_n = d(v_{n+1}, T(v_n, u_n, v_n)), \quad \gamma_n = d(w_{n+1}, T(w_n, v_n, u_n)).$$

Assume that $\lim_{n \rightarrow \infty} (\varepsilon_n, \delta_n, \gamma_n) = 0_{\mathbb{R}^3}$. Then

$$\begin{aligned} \varepsilon_n &= d(u_{n+1}, T(u_n, v_n, w_n)) = \left| u_{n+1} - \frac{2u_n - 2v_n + 2w_n + 1}{12} \right|, \\ \delta_n &= d(v_{n+1}, T(v_n, u_n, v_n)) = \left| v_{n+1} - \frac{2v_n - 2u_n + 2v_n + 1}{12} \right|, \\ \gamma_n &= d(w_{n+1}, T(w_n, v_n, u_n)) = \left| w_{n+1} - \frac{2w_n - 2v_n + 2u_n + 1}{12} \right|, \end{aligned}$$

and passing to the limit for $n \rightarrow \infty$, we obtain that

$$\lim_{n \rightarrow \infty} (u_n, v_n, w_n) = \left(\frac{1}{10}, \frac{1}{10}, \frac{1}{10}\right),$$

which is the unique tripled fixed point of T .

Hence, the tripled fixed point iterative procedure defined by (10) is T -stable.

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Тіміш І. *Стійкість ітераційних процедур для нерухомої точки третього порядку мішаних монотонних відображень* // Карпатські матем. публ. — 2014. — Т.6, №2. — С. 377–388.

Нещодавно Берінде і Боркут [11] ввели поняття нерухомої точки третього порядку і зараз вже є декілька досліджень цього об'єкту в частково впорядкованих метричних просторах і в конусоподібних метричних просторах.

У цій статті визначено поняття стійкості ітераційної процедури нерухомої точки третього порядку і отримані умови стійкості для мішаних монотонних відображень, які задовольняють різні умови стиску. Ці результати розширюють і доповнюють деякі відомі результати. Також подано ілюстративний приклад.

Ключові слова і фрази: нерухома точка третього порядку, стійкість, мішаний монотонний оператор, умови стиску.

Тимиш И. *Стойкость итерационных процедур для неподвижной точки третьего порядка смешанных монотонных отображений* // Карпатские матем. публ. — 2014. — Т.6, №2. — С. 377–388.

Недавно Беринде и Боркут [11] ввели понятие неподвижной точки третьего порядка и сейчас уже есть несколько исследований этого объекта в частично упорядоченных метрических пространствах и конусоидальных метрических пространствах.

В этой статье определено понятие стойкости итерационной процедуры неподвижной точки третьего порядка и получены условия стойкости для смешанных монотонных отображений, которые удовлетворяют разные условия сжатия. Эти результаты расширяют и дополняют некоторые известные результаты. Также приведён иллюстративный пример.

Ключевые слова и фразы: неподвижная точка третьего порядка, стойкость, смешанный монотонный оператор, условия сжатия.