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ON SOME GENERALIZATIONS OF BAER'S THEOREM

In this paper we obtained new automorphic analogue of Baer's theorem for the case when an arbitrary subgroup $A \leq Aut(G)$ includes a group of inner automorphisms $Inn(G)$ of a group G and the factor-group $A/Inn(G)$ is co-layer-finite.

Key words and phrases: A-center, A-commutator subgroup, co-layer-finite group, Baer's theorem.

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INTRODUCTION

Let G be a group and A be a subgroup of $Aut(G)$. The automorphism group A defines some standard subgroups of G. Among these subgroups one of the most well-known are

$$C_G(A) = \{g \in G | \alpha(g) = g, \forall \alpha \in A\},$$
$$[G, A] = \langle g^{-1}\alpha(g) = [g, \alpha] | g \in G, \alpha \in A\rangle.$$

We note that in general $C_G(A)$ is not normal in G, but if $Inn(G) \leq A$, then $C_G(A) \leq C_G(Inn(G)) = \zeta(G)$. In particular, $C_G(A)$ is a normal subgroup of G. Clearly $C_G(A)$ is A-invariant. The subgroup $C_G(A)$ is called the *A-center of G*.

On the other hand, a subgroup $[G, A]$ is normal for every subgroup $A \leq Aut(G)$. In fact, let $g, x \in G$, $\alpha \in A$, and consider $x^{-1}[g, \alpha]x$. We have

$$\begin{aligned} x^{-1}[g,\alpha]x =& x^{-1}g^{-1}\alpha(g)x = (gx)^{-1}\alpha(g)x = (gx)^{-1}\alpha(gxx^{-1})x = (gx)^{-1}\alpha(gx)\alpha(x^{-1})x \\ =& [gx,\alpha](\alpha(x))^{-1}x = [gx,\alpha](x^{-1}\alpha(x))^{-1} = [gx,\alpha][x,\alpha]^{-1} \in [G,A]. \end{aligned}$$

The subgroup $[G, A]$ is called the *A-commutator subgroup of G*.

Denote by ι_x the inner automorphism, defined by element x, that is $\iota_x(g) = x^{-1}gx$ for each $g \in G$. If $A = Inn(G)$, then A-center of G coincides with usual center of G and A-commutator subgroup coincides with the derived subgroup of G.

I. Schur was the first to study the relationships between the derived subgroup and the central factor-group in finite groups [7]. In his paper I. Schur investigated the so-called *Schur multiplicator* (all definitions see, for example, in the book [4, p. 14]). The following result was proved: *if G is a finite group then $[G, G] \cap \zeta(G)$ is isomorphic to a subgroup of $M(G/\zeta(G))$.* Here $M(H)$ denotes the Schur multiplicator of a group H. Later the construction of Schur multiplicator has been extended for arbitrary groups.

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R. Baer studied the case of infinite groups [1]. He proved that if $G/\zeta(G)$ is finite, then $[G, G]$ is also finite. But many mathematicians refer to this theorem as *Schur's theorem*. In connection with this result, the following question arises: *is there a function f such that $|[G, G]| \leq f(t)$ where $t = |G/\zeta(G)|$?* J. Wiegold has obtained here the best result. He proved that if $t = |G/\zeta(G)|$, then $|[G, G]| \leq w(t) = t^m$ where $m = \frac{1}{2}(log_p t - 1)$ and p is the smallest prime divisor of t [8, p. 347]. In the same paper J. Wiegold proved that this bound is attained if and only if $t = p^n$ where p is a prime [8, p. 347]. When t has more than one prime divisor the picture is less clear.

There are some distinct approaches for obtaining generalizations of the above theorem. One possibility is to use the automorphism groups. P. Hegarty in his paper [3, p. 929] proved that if $A = Aut(G)$ and $G/C_G(A)$ is finite, then $[G, A]$ is also finite. The condition $A = Aut(G)$ is very strong. The finiteness of $G/C_G(Aut(G))$ in this case implies that $Aut(G)$ is finite. In [2] a more general situation was considered: $Inn(G) \leq A$ and $A/Inn(G)$ is finite.

In the following we will show that it is not possible to extend the main results from [2, 3] on arbitrary automorphism group A. The following simple example shows this.

Let p be a prime, $G = \langle a \rangle \times K$, $|a| = p$ and $K = Dr_{n \in \mathbb{N}} \langle b_n \rangle$ be an elementary abelian p-subgroup. Then G has an automorphism α_j such that $\alpha_j(a) = ab_j$, $\alpha_j(x) = x$ for each $x \in K$. It is easily seen that every automorphism α_j has order p and a subgroup A of $Aut(G)$ generated by $\{\alpha_j | j \in \mathbb{N}\}$ is an elementary abelian p-group. Furthermore, $C_G(A) = K = [G, A]$ so that the factor-group $G/C_G(A)$ is finite, but the subgroup $[G, A]$ is infinite.

In this example an automorphism group A is bounded and infinite. Therefore it is natural to consider here the automorphism groups, which have unbounded and infinite factor-groups. One of such types of groups is following.

A group G is said to be *co-layer-finite* if the factor-group G/G^n is finite for all positive integers n. These groups were introduced in the paper [6, p. 500].

For such automorphism groups we obtained the following generalizations of the main results from [2, 3].

Theorem A. *Let G be a group and A be a subgroup of $Aut(G)$ such that $Inn(G) \leq A$. Suppose that $G/C_G(A)$ has finite order t. If $A/Inn(G)$ is co-layer-finite, then $[G, A]$ is a finite subgroup.*

The inclusion $Inn(G) \leq A$ implies that $C_G(A) \leq \zeta(G)$, and hence $Inn(G) \cong G/\zeta(G)$ is finite. Therefore the fact that the factor-group $A/Inn(G)$ is co-layer-finite implies that A is co-layer-finite.

R. Baer [1] obtained the following generalization of Schur's theorem.

Suppose that the term $\zeta_k(G)$ having a finite number k of the upper central series of a group G has finite index. Then the term $\gamma_{k+1}(G)$ of the lower central series of G is finite.

Starting from the A-center and A-commutator subgroups, we can define *the upper and the lower A-central series* of G. Suppose that $Inn(G) \leq A$. Put $\zeta_1(G, A) = C_G(A)$. If for ordinal ν we define $\zeta_\nu(G, A)$, then put $\zeta_{\nu+1}(G, A)/\zeta_\nu(G, A) = \zeta_1(G/\zeta_\nu(G, A), A/C_A(\zeta_\nu(G, A)))$. Shorter last group we will write as $\zeta_1(G/\zeta_\nu(G, A), A)$. As usual, if ν is a limit ordinal, then $\zeta_\nu(G, A) = \bigcup_{\mu<\nu} \zeta_\mu(G, A)$. The last term $\zeta_\gamma(G, A) = \zeta_\infty(G, A)$ of this series is called the *upper A-hypercenter of G*. The ordinal γ is called the *A-upper central length* of a group G, which we denote by $zl(G, A)$.

The lower A-central series of a group G is the series

$$G = \gamma_1(G, A) \geq \gamma_2(G, A) \geq \dots \gamma_\nu(G, A) \geq \gamma_{\nu+1}(G, A) \geq \dots \gamma_\delta(G, A)$$

defined by the rule $\gamma_2(G,A) = [G,A]$ and recursively $\gamma_{\nu+1}(G,A) = [\gamma_\nu(G,A),A]$ for all ordinals ν and $\gamma_\lambda(G,A) = \bigcap_{\mu<\lambda}\gamma_\mu(G,A)$ for the limit ordinals λ. The last term $\gamma_\delta(G,A) = \gamma_\infty(G,A)$ of this series is called the *lower A-hypocenter of G*.

An automorphic variant of the above Baer's theorem for the case when $A/Inn(G)$ is finite was obtained in [2]. The second main theorem of this paper gives a very wide generalization of this result.

Theorem B. *Let G be a group, A be a subgroup of $Aut(G)$ and Z be the upper A-hypercenter of G. Suppose that $Inn(G) \leq A$, $zl(G,A) = m$ is finite and G/Z is finite. If $A/Inn(G)$ is co-layer-finite, then $\gamma_{m+1}(G,A)$ is a finite subgroup.*

1 Preliminaries and lemmas

Let G be an abelian group, A be a subgroup of $Aut(G)$ and $\alpha \in A$. We define a mapping $d(\alpha) : G \to G$ by the rule $d(\alpha)(u) = u^{-1}\alpha(u) = [u,\alpha]$, $u \in G$. We have

$$d(\alpha)(uv) = (uv)^{-1}\alpha(uv) = v^{-1}u^{-1}\alpha(u)\alpha(v) = u^{-1}\alpha(u)v^{-1}\alpha(v) = d(\alpha)(u)d(\alpha)(v),$$

and then $d(\alpha)$ is an endomorphism of G. Furthermore, $Im(d(\alpha)) = [G,\alpha]$, $Ker(d(\alpha)) = C_G(\alpha)$. Thus we have $[G,\alpha] = Im(d(\alpha)) \cong G/Ker(d(\alpha)) = G/C_G(\alpha)$.

Let G be a finite group and suppose that $|G| = p_1^{k_1} \cdot \ldots \cdot p_n^{k_n}$ where $p_1 < \ldots < p_n$ are primes. Being nilpotent, the Sylow p_j-subgroup P_j of G has finite subnormal series whose factors are of prime order. It follows that for each $1 \leq j \leq m$ the Sylow p_j-subgroup P_j has at most k_j generators. It is not hard to see that $G = \langle P_1,\ldots,P_n\rangle$, so that G has at most

$$\begin{aligned} k_1+\ldots+k_n =& log_{p_1}|P_1| + \ldots + log_{p_n}|P_n| \leq log_{p_1}|P_1| + \ldots + log_{p_1}|P_n| \\ =& log_{p_1}(|P_1|\cdot\ldots\cdot|P_n|) = log_{p_1}|G| \leq log_2|G| \end{aligned}$$

generators.

Lemma 1. *Let G be an abelian group and A be a finite subgroup of $Aut(G)$. If $G/C_G(A)$ is a finite group, then $[G,A]$ is also finite and there exists a function δ such that $|[G,A]| \leq \delta(|G/C_G(A)|,|A|)$.*

Proof. Put $|G/C_G(A)| = t$, $|A| = k$ and $Z = C_G(A)$. As we have seen above $[G,\alpha] \cong G/C_G(\alpha)$ for every automorphism $\alpha \in A$. Since $Z \leq C_G(\alpha)$, $[G,\alpha]$ is a finite subgroup and $|[G,\alpha]| \leq t$.

Let $M = \{\alpha_1,\ldots,\alpha_d\}$ be a set of generators of A. As we have seen above, $d \leq log_2(|A|) = log_2(k)$. Since $|[G,\alpha_j]| \leq t$ for every $1 \leq j \leq d$ and $[G,A] = [G,\alpha_1]\cdot\ldots\cdot[G,\alpha_d]$ (see, for example, [5, Lemma 1.1]), $[G,A]$ is a finite and $|[G,A]| \leq td \leq tlog_2(k) = \delta(t,k)$. □

Let G be an abelian group, Z be a subgroup of G and C be a subgroup of $Aut(G)$ such that $C_C(Z) = C = C_C(G/Z)$. For arbitrary element g of G we defined the mapping $\eta_g : C \to Z$ by the rule $\eta_g(\alpha) = [g,\alpha]$, $\alpha \in C$. Since $\alpha \in C$, $\alpha(g) = gz$ for some element $z \in Z$, that is $z = g^{-1}\alpha(g) = \eta_g(\alpha)$. If β is another element of C, then

$$(\alpha\circ\beta)(g) = \alpha(\beta(g)) = \alpha(g\eta_g(\beta)) = \alpha(g)\alpha(\eta_g(\beta)) = g\eta_g(\alpha)\eta_g(\beta),$$

and so

$$\eta_g(\alpha\circ\beta) = g^{-1}(\alpha\circ\beta)(g) = g^{-1}g\eta_g(\alpha)\eta_g(\beta) = \eta_g(\alpha)\eta_g(\beta).$$

Therefore η_g is a homomorphism. We have $Ker(\eta_g) = C_C(g)$. Furthermore,

$$[g^2, \alpha] = g^{-2}\alpha(g^2) = g^{-2}\alpha(g)\alpha(g) = g^{-2}g\eta_g(\alpha)g\eta_g(\alpha) = (\eta_g(\alpha))^2.$$

Applying induction, we obtain that $[g^n, \alpha] = (\eta_g(\alpha))^n$. In particular, if the element gZ has finite order, that is there is a positive integer k such that $g^k \in Z$, then

$$(\eta_g(\alpha))^k = [g^k, \alpha] = g^{-k}\alpha(g^k) = g^{-k}g^k = 1.$$

In other words, $Im(\eta_g)$ is a subgroup of Z, having finite exponent $|g|$.

Suppose that the factor-group G/Z is periodic, then $\Pi(Im(\eta_g)) \subseteq \Pi(G/Z)$. We note that if $g \in Z$, then $\eta_g(\alpha) = 1$ for each $\alpha \in C$. The equation $\bigcap_{g\in G} C_C(g) = C_C(G) = \langle 1 \rangle$ together with Remak's theorem yields the embedding

$$C \hookrightarrow Cr_{g\in G} C/C_C(g) \cong Cr_{g\in G} Im(\eta_g).$$

The above inclusion $\Pi(Im(\eta_g)) \subseteq \Pi(G/Z)$ for each $g \in G$ shows that C is an abelian group such that $\Pi(C) \subseteq \Pi(G/Z)$.

Let G be a group and $A \leq Aut(G)$. Suppose that K is an A-invariant normal subgroup of G. For each automorphism $\alpha \in A$ we define the mapping $\alpha_K : G/K \to G/K$ by the rule $\alpha_K(xK) = \alpha(x)K$ for every $x \in G$. Clearly α_K is an endomorphism of G/K. Let $x \in G$ and $\alpha_K(xK) = K$, that is $K = \alpha(x)K$ and $\alpha(x) \in K$. Since K is A-invariant subgroup of G, $x \in K$ and $xK = K$. Therefore α_K is an automorphism of G/K. Furthermore, if $\alpha, \beta \in A$, then

$$(\alpha \circ \beta)_K(xK) = (\alpha \circ \beta)(x)K = \alpha(\beta(x))K = \alpha_K(\beta(x)K) = \alpha_K(\beta_K(xK)) = (\alpha_K \circ \beta_K)(xK).$$

Hence the mapping $\Phi : A \to Aut(G/K)$ given by $\Phi(\alpha) = \alpha_K$, $\alpha \in A$, is a homomorphism.

Lemma 2. *Let G be an abelian group and A be a subgroup of $Aut(G)$. Suppose that $G/C_G(A)$ is a finite group of order t. If A is co-layer-finite, then $[G, A]$ is a finite subgroup.*

Proof. Put $Z = C_G(A)$ and $C = C_A(G/Z)$. Clearly C is a normal subgroup of A. Since G/Z is finite, from above remarked we obtain that C is an abelian group of finite exponent. Since G/Z is finite, factor-group A/C is finite. Then Lemma 5 of the paper [6] implies that a subgroup C is co-layer-finite. Being abelian group of finite exponent, C must be finite. Hence $C = \{\gamma_1, \ldots, \gamma_s\}$ where $s = |C|$. As we saw above, for every automorphism $\alpha \in A$ we have an isomorphism $[G, \alpha] \cong G/C_G(\alpha)$. Since $Z \leq C_G(\alpha)$, $[G, \alpha]$ is a finite subgroup and $|[G, \alpha]| \leq t$. Since $[G, C] = [G, \gamma_1] \cdot \ldots \cdot [G, \gamma_s]$ (see, for example, [5, Lemma 1.1]), $[G, C]$ is a finite subgroup and $|[G, C]| \leq ts$.

Put $V = G/[G, C]$. We note that a subgroup $[G, C]$ is A-invariant. In fact, let g be an arbitrary element of G, $\alpha \in A$, $\gamma \in C$. Since C is a normal subgroup of A, $\alpha \circ \gamma = \gamma_1 \circ \alpha$ for some $\gamma_1 \in C$. Then

$$\begin{aligned}\alpha([g, \gamma]) =& \alpha(g^{-1}\gamma(g)) = \alpha(g^{-1})\alpha(\gamma(g)) = \alpha(g^{-1})(\alpha \circ \gamma)(g) = \alpha(g^{-1})(\gamma_1 \circ \alpha)(g) \\ =& \alpha(g^{-1})\gamma_1(\alpha(g)) = (\alpha(g))^{-1}\gamma_1(\alpha(g)) = [\alpha(g), \gamma_1] \in [G, C].\end{aligned}$$

Therefore $[G, C]$ is an A-invariant subgroup.

For each $\alpha \in A$ we define the mapping $\alpha_V : V \to V$ by the rule $\alpha_V(x[G, C]) = \alpha(x)[G, C]$ for every $x \in G$. Since $[G, C]$ is an A-invariant subgroup of G, by above remarked $\alpha_V \in Aut(V)$

and then there exists a homomorphism $\Phi : A \to Aut(V)$ given by $\Phi(\alpha) = \alpha_V$. Put $A_V = \Phi(A)$. If $\gamma \in C$, then $\gamma(g) = gg^{-1}\gamma(g) = g[g,\gamma]$, and hence

$$\gamma_V(g[G,C]) = \gamma(g)[G,C] = g[g,\gamma][G,C] = g[G,C]$$

for every $g \in G$. It follows that $C \leq Ker(\Phi)$. On the other hand, A/C is isomorphic to some subgroup of $Aut(G/Z)$. Since G/Z is finite of order t, A/C has finite order at most $t!$. Hence A_V is finite and $|A_V| \leq t!$. If $g \in Z$, then $\alpha_V(g[G,C]) = \alpha(g)[G,C] = g[G,C]$ for every $\alpha_V \in A_V$. It follows that $Z[G,C]/[G,C] \leq C_V(A_V)$. Thus $V/C_V(A_V)$ is a finite group and $|V/C_V(A_V)| \leq t$. For this case we can apply Lemma 1. By this Lemma $[V, A_V]$ is a finite subgroup, having order at most $\delta(t,t!)$. We have

$$[V, A_V] = [G/[G,C], A_V] = [G,A][G,C]/[G,C] = [G,A]/[G,C].$$

Therefore $[G,A]$ is a finite subgroup. □

2 PROOF OF THEOREM A

Proof. Put $|G/C_G(A)| = t$. Since $Inn(G) \leq A$, $C_G(A) \leq C_G(Inn(G)) = \zeta(G)$. It follows that $G/\zeta(G)$ is finite and $|G/\zeta(G)| \leq t$. Then $K = [G,G]$ is finite and $|[G,G]| \leq w(t) = t^m$ where $m = \frac{1}{2}(log_p t - 1)$ and p is the smallest prime divisor of t [8, p. 347].

Put $G_{ab} = G/K$. For each $\alpha \in A$ we define the mapping $\alpha_{ab} : G_{ab} \to G_{ab}$ by the rule $\alpha_{ab}(xK) = \alpha(x)K$ for every $x \in G$. Being characteristic subgroup of G, K is normal and A-invariant. As above $\alpha_{ab} \in Aut(G/K)$ and then there exists a homomorphism $\Phi : A \to Aut(G/K)$ given by $\Phi(\alpha) = \alpha_{ab}$. Put $A_{ab} = \Phi(A)$. Since G/K is abelian,

$$(\iota_g)_{ab}(x[G,G]) = \iota_g(x)[G,G] = g^{-1}xg[G,G] = x[x,g][G,G] = x[G,G]$$

for each $x \in G$. It follows that $Inn(G) \leq Ker(\Phi)$. In particular, A_{ab} is an epimorphic image of $A/Inn(G)$. If $x \in C_G(A)$, then $\alpha_{ab}(xK) = \alpha(x)K = xK$ for each $\alpha \in A$, which implies the inclusion $C_G(A)K/K \leq C_{G/K}(A_{ab})$. Hence $(G/K)/C_{G/K}(A_{ab})$ is a finite group and $|(G/K)/C_{G/K}(A_{ab})|$ is a divisor of $|G/C_G(A)|$. Applying Lemma 3 of the paper [6] the factor-group $A/Inn(G)$ is co-layer-finite. Then by the same Lemma 3 of the paper [6] its epimorphic image A_{ab} also is co-layer-finite. An application of Lemma 2 shows that $[G_{ab}, A_{ab}]$ has finite order. Furthermore,

$$[G_{ab}, A_{ab}] = [G/[G,G], A_{ab}] = [G,A][G,G]/[G,G].$$

If $g, x \in G$, then

$$[g,x] = g^{-1}x^{-1}gx = g^{-1}\iota_x(g) = [g,\iota_x].$$

It follows that $[G,G] \leq [G,A]$, because we have $Inn(G) \leq A$. Hence

$$[G_{ab}, A_{ab}] = [G,A]/[G,G].$$

And therefore $[G,A]$ is a finite subgroup. □

Let G be a group and H be a subgroup of G. Put $Inn_G(H) = \{\iota_x | x \in H\}$. Clearly $Inn_G(H)$ is a subgroup of $Inn(G)$.

3 PROOF OF THEOREM B

Proof. Let

$$\langle 1\rangle = Z_0 \leq Z_1 \leq \ldots \leq Z_{m-1} \leq Z_m = Z$$

be the upper A-central series of G. Put $|G/Z| = t$. We proceed by induction on m. If $m = 1$, then $Z_1 = C_G(A)$ has index at most t in G. Application of Theorem A shows that $[G, A] = \gamma_2(G, A)$ is finite.

Suppose inductively that the result is true for some integer $m > 1$ and let G be a group satisfying the hypotheses of the theorem with $zl(G, A) = m$. Consider the factor-group $L = G/Z_1$. Then

$$\langle 1\rangle = Z_1/Z_1 \leq \ldots \leq Z_{m-1}/Z_1 \leq Z_m/Z_1$$

is the upper A-central series of G/Z_1. For each $\alpha \in A$ we define the mapping $\alpha_L : L \to L$ by the rule $\alpha_L(xZ_1) = \alpha(x)Z_1$ for every $x \in G$. Since Z_1 is A-invariant, by above noted α_L is an automorphism of L and then there exists a homomorphism $\Phi : A \to Aut(L)$ given by $\Phi(\alpha) = \alpha_L$. Put $A_L = \Phi(A)$. It is not hard to prove that $Z/Z_1 \leq \zeta_{m-1}(G/Z_1, A_L)$. Hence $(G/Z_1)/\zeta_{m-1}(G/Z_1, A_L)$ is a finite group and its order is a divisor of $|G/Z|$. Clearly $\Phi(A) = A_L$ is an epimorphic image of A. Furthermore, $Inn(G/Z_1) = \Phi(Inn(G))$, so that $A_L/Inn(L)$ is an epimorphic image of $A/Inn(G)$. Since the factor-group $A/Inn(G)$ is co-layer-finite, by Lemma 3 of the paper [6] its epimorphic image $A_L/Inn(L)$ also is co-layer-finite.

Since $zl(G/Z_1, A_L) = m - 1$, by induction hypothesis $K/Z_1 = \gamma_m(G/Z_1, A_L)$ is finite. Clearly $D = \gamma_m(G, A) \leq K$. At once we note, that D is A-invariant. For every $\alpha \in A$ its restriction $\alpha|_D$ is an automorphism of D and the mapping $\Xi : A \to Aut(D)$ defined by the rule $\Xi(\alpha) = \alpha|_D$ is a homomorphism of A in $Aut(D)$. Moreover, $Ker(\Xi) = C_A(D)$. By Lemma 2 of the paper [2] $[Z, D] = \langle 1\rangle$, so that $Inn_G(Z) \leq Ker(\Xi)$. It follows that $\Xi(Inn(G))$ is an epimorphic image of G/Z, therefore $\Xi(Inn(G))$ is finite and has order at most t. The factor-group $\Xi(A)/\Xi(Inn(G))$ is an epimorphic image of $A/Inn(G)$, so that by Lemma 3 of the paper [6] $\Xi(A)/\Xi(Inn(G))$ is co-layer-finite. Since $|\Xi(Inn(G))| \leq t$, Lemma 4 of the paper [6] shows that $\Xi(A)$ is co-layer-finite. Clearly

$$Inn(D) = \Xi(Inn_G(D)) \leq \Xi(Inn(G)) \leq \Xi(A).$$

The inclusion $Z_1 \cap D \leq C_D(\Xi(A))$ shows that $D/C_D(\Xi(A))$ is a finite group and its order is at most $|K/Z_1|$. Hence we can apply Theorem A. The application of Theorem A shows that

$$[D, \Xi(A)] = [D, A] = [\gamma_m(G, A), A] = \gamma_{m+1}(G, A)$$

is finite. □

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В цій статті ми отримали новий автоморфний аналог теореми Бера у випадку, коли довільна підгрупа $A \leq Aut(G)$ містить групу внутрішніх автоморфізмів $Inn(G)$, а фактор-група $A/Inn(G)$ ко-шарово-скінченна.

Ключові слова і фрази: A-центр, A-комутаторна підгрупа, ко-шарово-скінченна група, теорема Бера.

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В этой статье нами был получен новый автоморфный аналог теоремы Бэра в случае, когда произвольная подгруппа $A \leq Aut(G)$ содержит группу внутренних автоморфизмов $Inn(G)$ группы G, а фактор-группа $A/Inn(G)$ ко-слойно-конечна.

Ключевые слова и фразы: A-центр, A-коммутаторная подгруппа, ко-слойно-конечная группа, теорема Бэра.